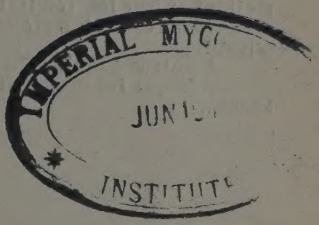
COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1945



VOLUME XLIX

№ 7

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

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NOUVELLE SÉRIE

13-me année

Paraissant tous les dix jours

1945

VOLUME XLIX, № 7

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Édition de l'Académie des Sciences de l'URSS
Moscou 1945

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

MATHÉMATIQUES

SUR UNE CLASSE D'ESPACES LINÉAIRES

Par M. M. GRINBLUM

(Présenté par A. N. Kolmogoroff, de l'Académie, le 7. II. 1945)

§ 1. Soit E un espace linéaire normé complet et séparable, et soit $\{x_i\}$ une base dans cet espace. Alors chaque élément $x \in E$ peut être présenté d'une manière unique dans la forme $x = \sum_{i=1}^{\infty} F_i(x)\, x_i$, et chaque fonctionnelle linéaire F, définie sur E, dans la forme

$$F(x) = \sum_{i=1}^{\infty} F_i(x)\, F(x) \tag{1}$$

Si la série (1) converge absolument pour chaque $x \in E$, nous dirons que $\{x_i\}$ est une base absolue.

Soit A une multiplicité linéaire de suites numériques $a \equiv \{a_i\}$. Si $a' \equiv \{a_i'\}$ et $a'' \equiv \{a_i''\}$, λ', λ'' étant des nombres, on a $\lambda' a' + \lambda'' a'' \equiv \{\lambda' a_i' + \lambda'' a_i''\}$.

Dans la suite nous allons toujours supposer que la multiplicité A contient toutes les suites de la forme $\eta_{ln} \equiv \{\underbrace{0, 0, \ldots, 0}_{n-1}, 1, 0\ 0, \ldots\}$. Convenons d'appeller ces η_{ln} — suites unitaires ou bien éléments unitaires.

Désignons ensuite par B la multiplicité linéaire des suites $b \equiv \{b_i\}$, déterminée de la manière suivante: $b \in B$ dans le cas et dans ce cas seulement où les séries $\sum_1^{\infty} a_i b_i$ convergent pour tous les $a \in A$; et par C — la multiplicité linéaire des suites telles que $c \in C$ dans le cas et dans ce cas seulement où les séries $\sum_1^{\infty} b_i c_i$ convergent pour tous les $b \in B$.

Lemme. Si l'on peut définir pour la multiplicité A une norme de telle manière que A devienne un espace de Banach régulier $\mathfrak{A}$, où les éléments unitaires η_{ln} forment une base, alors $A \equiv C$.

§ 2. Soit E un espace linéaire normé et complet (un espace de Banach), et A — une multiplicité de suites numériques pour laquelle on peut définir une norme de telle manière que A devienne un espace de Banach $\mathfrak{A}$ jouissant des propriétés suivantes:

(α) $\mathfrak{A}$ est un espace régulier. La norme d'un élément $a \in \mathfrak{A}$ sera désignée par $|a|_{\mathfrak{A}} = |\{a_i\}|_{\mathfrak{A}}$.

(β) Les éléments $\eta_{ln} = \{\underbrace{0, 0, \ldots, 0}_{n-1}, 1, 0, 0, \ldots\}$ $(n = 1, 2, \ldots)$ forment une base absolue dans $\mathfrak{A}$, et d'ailleurs $0 < \inf_n \{|\eta_{ln}|_{\mathfrak{A}}\} \leqslant \sup_n \{|\eta_{ln}|_{\mathfrak{A}}\} < \infty$.

(γ) Si $a' \equiv \{a_i'\}$ et $a'' \equiv \{a_i''\}$, et si d'ailleurs $0 \leqslant a_i' \leqslant a_i''$ pour tous les i, on a $|a'|_{\mathfrak{A}} \leqslant |a''|_{\mathfrak{A}}$.

(γ') Si $b' \equiv \{b'_i\}$ et $b'' \equiv \{b''_i\}$ sont deux éléments de l'espace $\overline{\mathfrak{A}}$, et $0 \leqslant b'_i \leqslant b''_i$ $(i = 1, 2, \ldots)$, on a $|b'|_{\overline{\mathfrak{A}}} \leqslant |b''|_{\overline{\mathfrak{A}}}$. On désigne ici par $|b|_{\overline{\mathfrak{A}}}$ la norme de l'élément $b \equiv \{b_i\}$ de l'espace $\overline{\mathfrak{A}}$.

Désignons par E_A la multiplicité de toutes les suites $\xi \equiv \{x_i\}$ d'éléments x de l'espace E, pour lesquels les suites correspondantes $\{\|x_i\|\}$ appartiennent à A.

Théorème 1. *Si l'on définit sur la multiplicité E_A une norme, en posant*

$$\text{norme } \xi = |\{\|x_i\|\}|_{\mathfrak{A}}$$

on aura: 1° *E_A devient un espace linéaire normé et complet (dans la suite nous désignerons cet espace par le symbole $E_{\mathfrak{A}}$).* 2° *Si E est séparable, alors $E_{\mathfrak{A}}$ est encore séparable.*

Théorème 2. 1° *A chaque fonctionnelle linéaire Φ déftnie sur $E_{\mathfrak{A}}$ il correspond — et d'une manière unique — une suite*

$$F_1^{\Phi},\ F_2^{\Phi},\ \ldots.\ F_n^{\Phi},\ \ldots$$

de fonctionnelles linéaires définies sur E, telle que la suite $\{|F_n^{\Phi}|\}$ est contenue dans $\overline{\mathfrak{A}}$. On a la relation

$$\Phi(x) = F_1^{\Phi}(x_1) + F_2^{\Phi}(x_2) + \ldots + F_n^{\Phi}(x_n) + \ldots$$

Inversement, chaque suite de fonctionnelles linéaires $F_1, F_2, \ldots, F_n, \ldots$ définies sur E et pour laquelle la suite numérique $\{|F_n|\}$ est contenue dans $\overline{\mathfrak{A}}$, définit sur $E_{\mathfrak{A}}$ une fonctionnelle linéaire Φ (et d'ailleurs on a $F_n = F_n^{\Phi}$).

2° *La norme $|\Phi|_{(\overline{E_{\mathfrak{A}}})}$ de la fonctionnelle Φ est définie par la formule*

$$|\Phi|_{(\overline{E_{\mathfrak{A}}})} = \sup_{|\{a_n\}|_{\mathfrak{A}} \leqslant 1,\ a_n \geqslant 0\ \ (n=1, 2, \ldots)} \sum_{1}^{\infty} |F_n^{\Phi}|\, a_n$$

c'est-à-dire la norme de Φ est égale à $\sup \sum_{n=1}^{\infty} |F_n^{\Phi}|\, a_n$ sur l'ensemble des éléments de la sphère unitaire de l'espace $\mathfrak{A}$ dont les coordonnées sont non négatives.

3° *Il existe une constante Q qui ne dépend pas de Φ et telle que*

$$|\{|F_n^{\Phi}|\}|_{\overline{\mathfrak{A}}} \leqslant Q\, |\Phi|_{(\overline{E_{\mathfrak{A}}})}$$

Théorème 3. *Si $\mathfrak{A}$ est l'espace des suites numériques vérifiant les conditions* (α), (β), (γ) *et* (γ'), *il existe une constante Q telle que*

$$\sup_{|\{a_i\}|_{\mathfrak{A}} \leqslant 1} \left| \sum_{i=1}^{\infty} b_i\, a_i \right| \leqslant Q \sup_{|\{a_i\}| \leqslant 1,\ a_i \geqslant 0\ (i=1, 2, \ldots)} \sum_{i=1}^{\infty} b_i\, a_i$$

pour toutes les suites $\{b_i\} \in \mathfrak{A}$ telles que $b_i \geqslant 0$ $(i = 1, 2, \ldots)$.

Théorème 4. *Si E est un espace régulier, $E_{\mathfrak{A}}$ est aussi un espace régulier.*

§ 3. Les propositions que nous venons de démontrer peuvent être appliquées à l'étude d'un système infini d'équations fonctionnelles linéaires d'une forme très générale à une infinité d'éléments inconnus. A savoir, le problème de l'existence d'une solution pour un système à une infinité d'éléments inconnus considéré pour l'espace E, se réduit d'une manière naturelle au problème concernant un système à un seul élément inconnu dans l'espace $E_{\mathfrak{A}}$.

Considérons le système d'équations

$$F_{\alpha 1}(x_1) + F_{\alpha 2}(x_2) + \ldots + F_{\alpha n}(x_n) + \ldots = C_{\alpha} \qquad (2)$$

Dans ce système $F_{\alpha n}$ sont des fonctionnelles linéaires définies sur E; x_n sont des éléments inconnus de l'espace E; C_{α} sont des nombres donnés. L'indice α parcourt un certain ensemble de valeurs Ω, en général, non dénombrable.

Théorème 5. *Si:* 1° *E est un espace régulier,* 2° *pour chaque valeur* $\alpha \in \Omega$ *la suite* $\{|F_{\alpha 1}|, |F_{\alpha 2}|, \ldots, |F_{\alpha n}| \ldots\}$ *appartient à l'espace* $\overline{\mathfrak{A}}$, *pour qu'il existe une suite* $x_1, x_2, \ldots$ *d'éléments de l'espace E vérifiant le système* (2) *et pour laquelle la suite* $\{\|x_n\|\}$ *soit un élément de l'espace* $\mathfrak{A}$, *il faut et il suffit qu'il existe une constante M telle que*

$$\left|\sum_{j=1}^{n} \lambda_j C_{\alpha_j}\right| \leqslant M \left|\left\{\left|\sum_{j=1}^{n} \lambda_j F_{\alpha_{ji}}\right|\right\}\right|_{\overline{\mathfrak{A}}} \tag{3}$$

quels que soient les $\alpha_j \in \Omega$ $(j = 1, 2, \ldots n)$, *quels que soient les nombres* $\lambda_1, \lambda_2, \ldots, \lambda_n$, *et pour chaque valeur entière et positive de n.*

Si l'on prend pour E l'espace $L^{(p)}$ et pour $\mathfrak{A}$ l'espace $l^{(p)}$ $(p > 1)$, le système (2) prend la forme

$$\int_0^1 \varphi_1(x)\,\psi_{\alpha 1}(x)\,dx + \int_0^1 \varphi_2(x)\,\psi_{\alpha 2}(x)\,dx + \ldots + \int_0^1 \varphi_n(x)\,\psi_{\alpha n}(x)\,dx + \ldots = C_\alpha \quad (\alpha \in \Omega)$$

Nous avons ici pour chaque $\alpha \in \Omega$

$$\sum_{i=1}^{\infty} \int_0^1 |\psi_{\alpha 1}(x)|^q\,dx < \infty \quad \left(\frac{1}{p} + \frac{1}{q} = 1\right)$$

Les fonctions inconnues doivent vérifier les inégalités

$$\sum_{n=1}^{\infty} \int_0^1 |\varphi_n(x)|^p\,dx < \infty$$

Dans ce cas la condition (3) prend la forme

$$\left|\sum_{j=1}^{n} \lambda_j C_{\alpha_j}\right| \leqslant M \left[\sum_{i=1}^{\infty} \int_0^1 \left|\sum_{j=1}^{n} \lambda_j \psi_{\alpha_j i}(x)\right|^q dx\right]^{1/q}$$

Il est facile de voir qu'on peut réduire au système (2) un système d'équations intégrales linéaires

$$\int_0^1 K_{\alpha_1}(x, s)\,\varphi_1(s)\,ds + \int_0^1 K_{\alpha_2}(x, s)\,\varphi_2(s)\,ds + \ldots + \int_0^1 K_{\alpha_n}(x, s)\,\varphi_n(s)\,ds + \ldots = f_\alpha(x)$$

où pour chaque $\alpha \in \Omega$ et pour chaque $x \in [0, 1]$ on a l'inégalité

$$\sum_{n=1}^{\infty} \int_0^1 |K_{\alpha_n}(x, s)|^q\,ds \leqslant \infty$$

La condition nécessaire et suffisante pour l'existence d'une suite $\{\varphi_i(x)\}$ vérifiant ce système et l'inégalité

$$\sum_{i=1}^{\infty} \int_0^1 |\varphi_i(x)|^p\,dx < \infty \quad \left(\frac{1}{p} + \frac{1}{q} = 1\right)$$

consiste dans l'existence d'une constante M telle que

$$\left|\sum_{j=1}^{n} \lambda_j f_{\alpha_j}(x_j)\right| \leqslant M \left[\sum_{i=1}^{\infty} \int_0^1 \left|\sum_{j=1}^{n} \lambda_j K_{\alpha_j i}(x_j, s)\right|^q ds\right]^{1/q}$$

pour chaque entier positif n, pour tous les $x_1, x_2, \ldots, x_n$ situées sur $[0, 1]$, et quels que soient les nombres $\lambda_1, \lambda_2 \ldots, \lambda_n$ et $\alpha_1, \alpha_2, \ldots, \alpha_n$ appartenant à Ω.

Université d'État
à Tachkent.

Manuscrit reçu
le 23. I. 1945.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

MATHEMATICS

ON NON-DECOMPOSIBILITY INTO A TOPOLOGICAL PRODUCT

By V. A. EPHRÄMOWITSCH

(Communicated by A. N. Kolmogoroff, Member of the Academy, 4. I. 1945)

The aim of the present note is to establish a sufficient criterion of non-decomposibility into a topological product.

More than eight years ago A. Kolmogoroff told me a very simple proof of non-decomposibility of the n-dimensional sphere S^n. This proof which has never been published, uses a property of the sphere S^n, owing to which it cannot be carried into a point by a continuous transformation on itself. If, however, one point c is removed from the sphere, the remainder can be carried into a point by a continuous transformation on itself.

Suppose that $S^n = XY$, where each of the factors contains more than one point; then every «coordinate parallel» (for instance Xy_1) will be transformed by the indicated deformation of the remainder $S^n - c$ into a point $c_0 = x_0 y_0$ (we assume that $c = ab$ is chosen outside of this parallel $(b \neq y_1)$).

Thus, there exists a continuous deformation of the parallel transforming it on the sphere into the point (x_0, y_0). Projecting this last deformation onto the corresponding factor «parallel» to the other factor $(\mathrm{Pr}_X xy = x)$ we obtain a continuous deformation of the first factor on itself transforming it into the point (x_0) (X is homotopic to zero in X, Y is homotopic to zero in Y).

Subjecting the two factors simultaneously to such a deformation $(x \to x_0,\ y \to y_0)$, we thereby define a continuous deformation of the whole product XY on itself into a point $(xy \to x_0 y_0)$, but this contradicts the property of the sphere mentioned above.

I have never met in literature with proofs of non-decomposibility of other manifolds, not to speak of criterions. The following criterion may be therefore of some interest, inspite of the fact that the considerations leading up to it are of a very elementary nature.

Criterion of non-decomposibility. If the polynomial of Poincaré $P(R)$ of a normal topological space R of dimensionality n is irreducible in the domain of non-negative integers, and the senior Betti number $p_n(R) > 0$, then R cannot be decomposed into a topological product.

Under p_k, $k = 0, 1, \ldots, n$, are to be understood the Betti numbers corresponding to the ∇-groups of Betti in the sense of Alexandroff [1] by any simple modulus or by modulus zero, and under $P(R)$ the corresponding polynomial of Poincaré: $P(R) = \sum_0^n p_k t^k$.

Proof*. Let the condition be satisfied and yet $R = XY$, where X and Y are general topological spaces ([2]). It is easily seen that X and Y have a denumerable basis also and are normal. Since the Poincaré polynomials are multiplicative (for Betti numbers by a field of integers cf. ([3])), $P(X)P(Y) = P(XY) = P(R)$. By assumption $P(R)$ is irreducible, and hence one of the factors, for instance $P(Y)$, is equal to one, and consequently $P(X) = P(R)$ and $p_n(X) = p_n(R) > 0$. Hence follows that $\dim X \geqslant n$, and therefore $\dim Y = 0$, since otherwise ([4]) it would follow that $\dim(XY) > n$. Since now $\dim Y = 0$, $p_k(Y) = 0$ for $k \geqslant 1$ and $P(Y) = p_0(Y)$. Thus, $p_0(Y) = 1$ and, consequently, Y consists of only one point, *i. e.* R admits of a trivial decomposition only.

Applying the criterion to an orientable manifold (or pseudomanifold) we conclude that the non-decomposibility is guaranteed here by the irreducibility of the Poincaré polynomial, and for non-orientable manifolds (and pseudomanifolds) by the irreducibility of the Poincaré polynomial by the modulus 2.

In particular, hence follows the non-decomposibility of closed surfaces except the thorus and Klein's surface. But the latter is also non-decomposible, as may be proved in a different way, whereas the thorus is the topological product of two circumferences.

Received
1.I.1945.

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* The proof is stated here in a modified version suggested by a remark of P. Alexandrov and L. Pontrjagin. In consequence of this remark the criterion itself has received a more general form.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

MATHEMATICS

THE GENERAL CASE OF LOCATING A POINT ON A PLANE BY THREE ANGLE MEASUREMENTS

By M. I. YUDIN

(Communicated by V. I. Smirnov, Member of the Academy, 17. II. 1945)

An earlier paper by the author ([1]) contains an analytical solution of the problem of location of a point with the least mean square error by three angle measurements. It also contains the expression for the minimum of this error through the mean squares δ_j^2 of the angular errors δ_j of the separate measurements. The errors δ_j were considered as small independent random variables subject to the normal law of distribution. In a number of practically important cases, however, the errors δ_j cannot be regarded as independent, since they all arise from one source. Such is the case with aerodynamical displacement or the error involved in adjusting sights when the wind is determined by the three angles of displacement. The error involved in the determination of the sound wind in direction finding is another example, and there are many more similar cases. It turns out that errors of this kind, which are systematical errors for a given series of measurements, do not essentially affect the rule of location of a point with the highest possible accuracy. We will establish the solution of the problem in the general case.

Let the true azimuths of directions to the point O from the points P_1, P_2, P_3 be $\alpha_1-\delta_1$, $\alpha_2-\delta_2$, $\alpha_3-\delta_3$, respectively. We denote the «error triangle» arising from the errors of measurements by ABC.

We fix the origin of coordinates somewhere outside of our plane and denote the radii-vectors of the points O, A, B, C by $\vec{r}_0$, $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, respectively, and the vectors $\overrightarrow{AO}$, $\overrightarrow{BO}$, $\overrightarrow{CO}$ by $\vec{l}_1$, $\vec{l}_2$, $\vec{l}_3$. Then, as we know, the radius-vector $\vec{r}$ of an arbitrary point N lying in the plane ABC is representable uniquely in the form

$$\vec{r} = \lambda_1 \sin\beta_1 \vec{r}_1 + \lambda_2 \sin\beta_2 \vec{r}_2 + \lambda_3 \sin\beta_3 \vec{r}_3 \tag{1}$$

where

$$\beta_1 = \alpha_2 - \alpha_3, \quad \beta_2 = \alpha_3 - \alpha_1, \quad \beta_3 = \alpha_1 - \alpha_2$$

The coefficients λ_j may be expressed by the following simple formula involving the radius R of the circle circumscribed about the triangle ABC, the area of the triangle F and the distance d_j from the corresponding side of the triangle (taken with the proper sign)

$$\lambda_j = \frac{R}{F} d_j \tag{2}$$

The λ_j are connected with each other by the relation

$$\lambda_1 \sin\beta_1 + \lambda_2 \sin\beta_2 + \lambda_3 \sin\beta_3 = 1 \tag{3}$$

Subtracting from the left and right-hand side of the equality (1) the vector $\vec{r}_0$, we find

$$\vec{r}-\vec{r}_0=\lambda_1\sin\beta_1(\vec{r}_1-\vec{r}_0)+\lambda_2\sin\beta_2(\vec{r}_2-\vec{r}_0)+\lambda_3\sin\beta_3(\vec{r}_3-\vec{r}_0)$$

Expressing $\vec{r}-\vec{r}_0$ through the angular errors δ and retaining only the first powers of the small quantities δ_j, we obtain

$$\begin{aligned}\vec{r}-\vec{r}_0=&\lambda_1\left(l_3\delta_3\frac{\vec{l}_2}{l_2}-l_2\delta_2\frac{\vec{l}_3}{l_3}\right)+\\&+\lambda_2\left(l_1\delta_1\frac{\vec{l}_3}{l_3}-l_3\delta_3\frac{\vec{l}_1}{l_1}\right)+\\&+\lambda_3\left(l_2\delta_2\frac{\vec{l}_1}{l_1}-l_1\delta_1\frac{\vec{l}_2}{l_2}\right).\end{aligned}\qquad(4)$$

Our problem consists in finding such a point N_0 for which the mathematical expectation of the square of the distance from the true point O should be minimal

$$E(\vec{r}-\vec{r}_0)^2=\min$$

The mathematical expectations of the squares and the products of the angular errors must be considered to be known

$$E\delta_i\delta_j=B_{ij}\qquad(5)$$

and, in view of the presence of systematical errors, we can no longer assume

$$B_{ij}=0\quad\text{for}\quad i\neq j$$

In order to find the values λ_1, λ_2, λ_3 for which the square of the distance NO is minimum and which satisfy the condition (3) it is sufficient to equate to zero the derivatives by the λ_j of the expression

$$E(\vec{r}-\vec{r}_0)^2+2\mu(1-\lambda_1\sin\beta_1-\lambda_2\sin\beta_2-\lambda_3\sin\beta_3)$$

The application of the theorem on homogeneous functions at once discloses the meaning of the factor μ which is equal to the mean square of the distance N_0O. Further intermediary computations, the forming of the mean square of the vector $\vec{r}-\vec{r}_0$, its differentiation and the solution of the system of linear equations are omitted. It is essential, however, that direct calculation of the determinant of the system D can be dispensed with. In fact, the solution of the system of equations assumes the form

$$\lambda_k=\frac{\mu l_k}{D}\sum_{ij}B_{ij}l_il_j\sin\beta_i\sin\beta_j\sum_h B_{hk}l_h\sin\beta_h$$

(i, j, h, k take the values 1, 2, 3).

Substituting the obtained values in (3), we find

$$\frac{\mu}{D}\left(\sum_{ij}B_{ij}l_il_j\sin\beta_i\sin\beta_j\right)^2=1$$

Hence follows finally

$$\lambda_k=\frac{l_k\sum_h B_{hk}l_h\sin\beta_h}{\sum_{ij}B_{ij}l_il_j\sin\beta_i\sin\beta_j}\qquad(6)$$

after which any of the equations connecting λ_k and μ enables us to find the value of μ:

$$\mu=\frac{\frac{1}{2}\sum_{ij}(l_i^2B_{ii}l_j^2B_{jj}-l_i^2l_jB_{ij}^2)+\sum_i 2l_i^2l_jl_k\cos\beta_i(B_{ij}B_{ik}-B_{ii}B_{jk})}{\sum_{ij}B_{ij}l_il_j\sin\beta_i\sin\beta_j}\qquad(7)$$

where in the second sum in the numerator the indices i, j, k are not equal to each other.

The formulae (2) and (6) completely determine the position of the most probable point N_0, and the formula (7) may serve as a characteristics of the achieved accuracy. The quantities B_{ij} involved in the formula must be previously determined by a theoretical and experimental study of the errors of the employed method of measurement. Attempt to estimate the values of B_{ij} from the dimensions of the «error triangle» ABC may succeed only in exceptional cases where the triangle has practically inadmissible dimensions for the given method of measurements, which indicates the presence of a gross error. Small dimensions of the triangle, however, do not necessarily imply a high precision of measurements, as is all too often supposed. In solving a concrete problem it is not, of course, always possible to calculate the coordinates of the point N_0, and we have to develop some approximate rules based on the formulae (6) and (7). Let us illustrate this by a simplest example.

Let

$$B_{ij}=\begin{cases}\sigma^2+\Delta^2 & \text{for } i=j\\ \Delta^2 & \text{for } i\neq j\end{cases}$$

(Δ^2, the mean square of the systematical error of measurements),

$$l_1=l_2=l_3=l$$

Consider now two possible cases leading to an equilateral triangle ABC:

I. $\beta_1=\beta_2=\beta_3+2\pi=\frac{2\pi}{3}$

II. $\beta_1=\beta_3=\frac{\pi}{2}$; $\beta_2=-\frac{2\pi}{3}$

Then from the formulae (2) and (6) we obtain:

For the case I

$$\lambda_1=\lambda_2=\lambda_3=\frac{3\sqrt{3}}{4}R\,\frac{\frac{\sqrt{3}}{2}(\sigma^2+3\Delta^2)}{\frac{9}{4}(\sigma^2+3\Delta^2)}=\frac{R}{2}$$

i. e. whatever may be the systematical error, the most probable position of the point N_0 is at the centre of the triangle.

For the case II

$$\lambda_1=\lambda_3=\frac{3\sqrt{3}}{4}R\,\frac{\frac{\sqrt{3}}{2}(\sigma^2+\Delta^2)}{\frac{3}{4}(3\sigma^2+\Delta^2)}=\frac{3R}{2}\,\frac{\sigma^2+\Delta^2}{3\sigma^2+\Delta^2},$$

$$\lambda_2=\frac{3R}{2}\,\frac{\sigma^2-\Delta^2}{3\sigma^2+\Delta^2}$$

and already for $\Delta=\sigma$, $\lambda_2=0$ while for $\Delta>\sigma$, $\lambda_2<0$, *i. e.* the most probable point lies outside of the error triangle.

We see that in the last case it is very important to determine the relation between the systematical and random errors of the method of measurement with sufficient accuracy.

Below is set out a table of values of the quantity $E(\vec{r}-\vec{r}_0)^2/l^2(\sigma^2+\Delta^2)$ as a function of two parameters: θ which is the value of Δ/σ taken for the basis in constructing the rule of determination of the point N_0, and t which is the true value of Δ/σ.

Tables of Values $\frac{E(\overrightarrow{r} - \overrightarrow{r_0})^2}{l^2(\sigma^2 + \Delta^2)}$

θ \ t	0	1/2	1	2	∞
0	1.33	1.42	1.56	1.69	1.78
1/2	1.34	1.40	1.50	1.59	1.65
1	1.67	1.53	1.33	1.13	1.00
2	3.07	2.54	1.73	0.92	0.39
∞	6.67	5.33	3.33	1.33	0.00

The construction of such tables is very important for a propre choice of rules giving a sufficient accuracy in locating the point and is also convenient in practice. It may be remarked in conclusion that the question of the analytical construction of rules that take into account the nature of systematical errors does not appear to have ever been discussed before. And it may be hoped that the indicated approach to the solution of such questions may find a wide application in the theory of errors.

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Received
17. II. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

FLUID DYNAMICS

FLOW PAST A SYMMETRICAL AEROFOIL

By J. M. SEREBRIISKY

(Communicated by S. A. Khristianovich, Member of the Academy, 7. II. 1945)

This paper is concerned with a two-dimensional flow of an ideal incompressible fluid past a thin smooth symmetrical aerofoil placed at a zero angle of incidence.

Suppose that the aerofoil is given in a rectangular system of coordinates x, y. Let us pass in the plane of the aerofoil to the elliptical coordinates (φ, ϑ), taking into consideration that

$$x = \operatorname{ch}\psi \cos\theta, \quad y = \operatorname{sh}\psi \sin\theta \tag{1}$$

The velocity potential Φ satisfies Laplace's equation

$$\frac{\partial^2\Phi}{\partial\psi^2} + \frac{\partial^2\Phi}{\partial\theta^2} = 0 \tag{2}$$

Denote the potential of disturbed motion by Φ_1. In the case of a symmetrical flow equation (2) will be satisfied if we put

$$\Phi_1 = \sum_{n=1}^{\infty} A_n \exp(-n\psi)\cos n\theta$$

To obtain Φ we must add to Φ_1 the potential of uniform flow. Making use of the Cauchy-Rieman equation, we can introduce the stream function Ψ. At the surface of the body we have $\Psi = 0$, which means that the following relation should be satisfied at the surface

$$\Psi = -\sum_{n=1}^{\infty} A_n \exp(-n\psi)\sin n\theta + y = 0 \tag{3}$$

Denote the projections of the velocity onto the lines $\psi = \text{const}$ and $\theta = \text{const}$ by v_ψ and v_θ, respectively, where

$$v_\psi = \frac{1}{\sqrt{\operatorname{ch}^2\psi - \cos^2\theta}}\left[-\sum_{n=1}^{\infty} nA_n \exp(-n\psi)\cos n\theta + \operatorname{sh}\psi\cos\theta\right]$$

$$v_\theta = \frac{1}{\sqrt{\operatorname{ch}^2\psi - \cos^2\theta}}\left[-\sum_{n=1}^{\infty} nA_n \exp(-n\psi)\sin n\theta - \operatorname{ch}\psi\sin\theta\right] \tag{4}$$

The absolute value of the velocity $v = \sqrt{v_\psi^2 + v_\theta^2}$.

In the case of a thin aerofoil the values of ψ at its surface are small enough. It is only natural to make use of this circumstance. Represent $\exp(-n\psi)$ in the form of a series

$$\exp(-n\psi) = 1 - n\psi + \frac{n^2\psi^2}{2} - \cdots = 1 + \delta_n \tag{5}$$

Substitute $\exp(-n\psi) = 1 + \delta_n$ in (3) and suppose that the quantity δ_n may be neglected in the first approximation. We thus obtain

$$y = \sum_{n=1}^{\infty} A_n \sin n\theta \tag{6}$$

The solution of the problem is reduced to the expansion of y in a trigonometric series in θ.

Substitute the expansion of $\exp(-n\psi)$ in the formula for the determination of the velocity, omitting the quantity δ_n in first approximation. It may easily be shown that in this case we have $v_\psi = 0$, *i. e.* the velocity component normal to the ellipse drawn through the point under consideration vanishes.

At first sight it would seem that such an approximation may give rise to large errors for large dy/dx near the leading edge. But if we place the focus of the ellipse in the middle of the radius of curvature of the leading edge the outline near the latter will coincide with the ellipse and then diverge from it smoothly. So the error near the leading edge is eliminated by the introduction of the coordinates ψ, θ. Using expansion (6) we proceed to compute the velocity $v \simeq v_0$ by the formula

$$v = \frac{\operatorname{ch}\psi \sin\theta + \sum_{n=1}^{\infty} nA_n \sin n\theta}{\sqrt{\operatorname{ch}^2\psi - \cos^2\theta}} \tag{7}$$

Similar simplifications can be made in computing the velocity distribution in the case of a longitudinal flow past a body of revolution [1]. In fact, we have then

$$v = \frac{\operatorname{ch}\psi \sin\theta + \sum_{n=1}^{\infty} B_n Q_n(\lambda) \frac{dP_n}{d\mu}}{\sqrt{\operatorname{ch}^2\psi - \cos^2\theta}}, \quad \left(\psi = \sum_{n=1}^{\infty} \frac{2B_n}{n(n+1)} \frac{dP_n}{d\mu}\right) \tag{8}$$

(where $\lambda = \operatorname{ch}\psi$, $\mu = \cos\theta$, P_n are Legendre polynomials and the Q_n are Legendre functions of the second kind).

Computation has shown that for ordinary aerofoils the results obtained in first approximation are quite satisfactory in point of accuracy, and, if necessary, the correction for second approximation can easily be introduced.

At the surface of a thin aerofoil $\operatorname{ch}\psi = 1 + \psi^2/2 + \ldots$ is very close to unity. But near the leading and trailing edges $\cos\theta$ is likewise near to unity. Therefore putting $\operatorname{ch}\psi = 1$ in formula (7) will introduce great errors in the calculation of the velocity in the immediate vicinity of the leading and trailing edges, and the thinner the aerofoil, the smaller will be the error.

Setting $\operatorname{ch}\psi = 1$, we obtain

$$v = 1 + \sum_{n=1}^{\infty} \frac{nA_n \sin n\theta}{\sin\theta} \tag{9}$$

which connects the aerofoil coordinate $\left(y = \sum_{n=1}^{\infty} A_n \sin n\theta\right)$ with the velocity distribution and *vice versa*. In formula (9) we have the velocity of the undis-

turbed flow (unity) separated from the velocities (perturbation) induced by the presence of the body. Formula (9) is very useful for an approximate estimation of the aerofoil outline from a given distribution of velocities.

Let us see, for example, what form should we give a body in order to obtain the most uniform velocity distribution along the chord. For an elliptic cylinder ($y = A_1 \sin \vartheta$) we have, according to (9), $v = 1 + A_1$, *i. e.*, a constant velocity. An elliptic cylinder is the body that gives the least induced velocities for a given relative thickness. The principal distortion introduced by the formula (9) is that at the stagnation point the velocity is finite and distinct from zero. For that reason practical computations must be carried out by means of formula (7).

Received
7. II. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

THEORY OF ELASTICITY

GENERAL FORMULAE FOR THE REFLEXION AND REFRACTION OF NON-STATIONARY ELASTIC WAVES

By V. GOGOLADZE

(Communicated by N. N. Lusin, Member of the Academy, 4. XII. 1944)

From the results of an investigation into the problem of reflexion and refraction of waves (1) it appears that elastic disturbances may be considered as an aggregate of longitudinal and transversal waves reflected and refracted under real and complex angles in the plane of separation between elastic media; consequently, as an aggregate of homogeneous and non-homogeneous plane waves. In the case where the condition $R(b_2) > 0\ (b_2 < b_1)$ or the condition $R(b_1) > 0\ (b_1 < b_2)$ is satisfied, the boundary wave of Rayleigh is added to them, whose propagation velocity is smaller than that of all other waves. For the half-space general formulae of reflexion were obtained by S. Soboleff (2).

All results obtained by me may be boiled down to the following general formulae of reflexion and refraction of elastic waves:

$$\left.\begin{aligned}
\varphi_1 &= \operatorname{Re}\left\{ + R(p,\alpha) f_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) + \right. \\
&\left. + \sum_{j=1}^{4} l_j(p,\alpha) f_j\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right)\right\} \\
\psi_1 &= \operatorname{Re}\left\{ + R(p,\alpha) f_2\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right) + \right. \\
&\left. + \sum_{j=1}^{4} m_j(p,\alpha) f_j\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right)\right\}
\end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned}
\varphi_2 &= \operatorname{Re}\left\{ R(p,\alpha) f_3\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right) + \right. \\
&\left. + \sum_{j=1}^{4} n_j(p,\alpha) f_j\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right)\right\} \\
\psi_2 &= \operatorname{Re}\left\{ R(p,\alpha) f_4\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right) + \right. \\
&\left. + \sum_{j=1}^{4} p_j(p,\alpha) f_j\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right)\right\}
\end{aligned}\right\} \quad (2)$$

where l_j, m_j, n_j and p_j are coefficients obtained in solving the boundary equations satisfied on the plane of separation (3); f_j are functions regular in the upper complex half-plane; $R(p, \alpha)$ is the Rayleigh function.

These formulae contain all possible vibrations arising in the process of reflexion and refraction of plane waves. In fact, suppose, for instance, that the velocities of the longitudinal and transversal waves satisfy the inequality

$$b_2 < a_2 < b_1 < a_1$$

1. Let $|c| = |p/\alpha| > a_1$. Then in the formulae (1) and (2) f_j express homogeneous (real) reflected and refracted plane waves

2. If $b_1 < |c| < a_1$, then function f_1 has for argument the expression $pt + \alpha x \pm iy\sqrt{\alpha^2 - p^2/a_1^2}$. Hence, in virtue of the assumption made with respect to f_j, f_1 must be bounded in the whole plane of the complex variable, and so, by Liouville's theorem, must be a constant. This function may be therefore omitted in the formulae (1) and (2). It may be easily shown that the formulae (1) and (2) satisfy the boundary conditions of the problem without the function f_1.

The potentials φ_2, ψ_2 and ψ_1 represent homogeneous plane waves. As regards the potential φ_1, it represents a plane non-homogeneous (complex) wave. The phase velocity of the non-homogeneous wave along the oy-axis is an imaginary quantity. Thus, the disturbance decreases with increasing $y > 0$, although the medium considered does not absorb energy during this process.

3. In the interval $a_2 < |c| < b_1$ the function f_2 has for argument the expression $pt + \alpha x \pm iy\sqrt{\alpha^2 - p^2/b_1}$. Hence f_2 may be considered constant and can be eliminated from the formulae (1) and (2).

4. In the interval $b_2 < |c| < a_2$ the function f_3 has for argument the expression $pt + \alpha x \pm iy\sqrt{\alpha^2 - p^2/a_2^2}$, and the functions f_1, f_2 and f_3 must be considered constant.

5. Let, finally, $0 < |c| < b_2$. If $R(b_2) > 0$, then for $c = \pm\varepsilon$ [4] the coefficient of the first summand in the formulae (1) and (2) is zero; $R(\pm\varepsilon) = 0$.

In virtue of Rayleigh's equation the coefficients of the functions f_j are connected by some relations. Thus, into the expression of potentials enters a linear combination of the f_j. Without loss of generality we may put $f_2 = f_3 = f_4 = 0$. Hereby we obtain formulae expressing the boundary wave of Rayleigh.

We proceed now to investigate the nature of the non-homogeneous waves so far as energy is concerned. If the reflected and the refracted waves are homogeneous, then the energy flows of these waves have a constant direction coinciding with the normals to the fronts of plane waves, which are the rays along which the enegy is propagated. For the study of the energy motion of non-homogeneous waves we introduce, instead of the rays, the notion of the line of energy flow of elastic waves. Suppose, for instance, that in the medium $y < 0$ arises as the result of refraction a non-homogeneous longitudinal wave and a homogeneous transversal wave.

The front of the longitudinal non-homogeneous wave is determined by the equation

$$pt + \alpha x - iy\sqrt{\alpha^2 - p^2/a_2^2} = 0$$
$$\text{or } pt + \alpha x = 0, \quad y = 0 \tag{3}$$

The wave field of the longitudinal disturbance in $y < 0$ is determined by the formula

$$\varphi_2 = \operatorname{Re} A(p, \alpha) f\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right) \tag{4}$$

where the function f is regular in the upper complex half-plane, and $A(p, \alpha)$ is a complex number.

The elementary energy flow $\vec{S}(S_x, S_y)$ is determined by the formula [5]

$$\left.\begin{aligned} S_x &= -\mu_2 \left[\frac{\partial u}{\partial t} \tau_{xx} + \frac{\partial v}{\partial t} \tau_{xy} \right] \\ S_y &= -\mu_2 \left[\frac{\partial u}{\partial t} \tau_{xy} + \frac{\partial v}{\partial t} \tau_{yy} \right] \end{aligned}\right\} \tag{5}$$

Introduce the notation

$$A(p, \alpha) f'' \left(pt + \alpha x - iy \sqrt{\alpha^2 - \frac{p^2}{a_2^2}} \right) = \omega_1 + i\omega_2 \tag{6}$$

Substituting (4) into (5) and taking the real parts in (5), we obtain after easy transformations the following result

$$S_x = -\frac{\mu_2 \alpha}{b_2^2} \left[\omega_1^2 + 2b_2^2 \left(\alpha^2 - \frac{p^2}{a_2^2} \right) (\omega_1^2 + \omega_2^2) \right] \tag{7}$$

where

$$\alpha^2 - \frac{p^2}{a_2^2} > 0 \tag{8}$$

$$S_y = -\frac{\mu^2}{b_2^2} \sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\, \omega_1 \omega_2 . \tag{9}$$

From these formulae follows an important property of the flow of non-homogeneous waves. According to (7) and (9), the horizontal component of the energy flow S_x has a constant direction. The vertical component of the energy flow S_y may change its sign in the course of time and at different points.

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Received
4. XII. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

PHYSICS

RADIATION OF GRAVITATIONAL WAVES BY ELECTROMAGNETIC WAVES

By L. M. BREKHOVSKICH

(Communicated by S. I. Vavilov, Member of the Academy, 20. II. 1945)

1. Introduction. An electron moving uniformly in a medium is known to radiate electromagnetic waves if its velocity exceeds the phase velocity of propagation of the above waves in this medium. The phenomenon is described in electrodynamics by the name of «Cherenkov effect». A similar case is the emission of sound waves by a projectile moving with a speed higher than the velocity of sound.

It is highly probable that an effect of this kind may also exist in the general theory of relativity. In a world with a curvature distinct from zero (a non-Euclidean world) gravitational and electromagnetic waves propagate in the general case with a different speed. It may be supposed by analogy that a photon will radiate gravitational waves, thereby losing some of its energy, when it has outspeeded the rate of their propagation.

But the radiation of gravitational waves by electromagnetic waves is not confined to this case. It does also take place in a plane world when the two kinds of wave are moving with equal speed.

In 1937 Rosen([1]) showed that the joint system of gravitational and electromagnetic equations has no solution in the form of plane waves.

If such a solution were obtained, say, for the electromagnetic waves, it would give origin to infinitely increasing solutions of the gravitational equations. and thus yield an infinitely great disturbance in the metric of the world.

So there is no possibility left for the existence of plane electromagnetic waves. The plane wave, however, is always merely an idealization, and to deny its existence in our case is simply to say that no such idealization is possible in the general theory of relativity. The question arises, what will be the gravitational radiation of a spherical wave, for the existence of the latter is a reality. The solution of this question is given in the present paper, whereby is also settled the problem as to the existence of a phenomenon analogous to the Cherenkov effect for this form of wave.

2. Electromagnetic Wave. We shall consider the solution of electromagnetic field equations corresponding to the radiation of a variable dipole, leaving aside for the present any reference to gravitation.

Maxwell's equations in curvilinear coordinates are as follows

$$F^{ik}_{ik}=0; \quad \frac{\partial F_{ik}}{\partial x_l}+\frac{\partial F_{li}}{\partial x_k}+\frac{\partial F_{kl}}{\partial x_i}=0 \tag{1}$$

where F_{ik} is the tensor of the electromagnetic field.

For the field of the oscillator in the wave zone F_{13}, F_{23}, F_{43} correspond to H_ϑ, H_r, E_φ and are equal to zero. We further have $F_{21} = rH_\varphi$, $F_{24} = -irE_\vartheta$; $F_{14} = -iE_r$ is two orders of smallness less than F_{21} and F_{24}, and if $1/r$ is regarded as a small quantity, this component may be discarded as well.

We first confine ourselves to the case of Einstein's cylindrical world whose metric in spherical coordinates r, ϑ, φ is

$$-ds^2 = \frac{dr^2}{1-\eta^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) + dx_4^2 \tag{2}$$

where $x_4 = ict$, $\eta = r/a$.

With the radius of the world $a \to \infty$ (2) becomes the metric of the Euclidean world. Deciphering the field equations (1) for a space with the metric (2) we obtain

$$\frac{\partial F_{12}}{\partial x_4} + \frac{\partial F_{24}}{\partial r} = 0; \quad \frac{\partial}{\partial r}(\sqrt{1-\eta^2}F_{12}) = \frac{1}{\sqrt{1-\eta^2}}\frac{\partial F_{24}}{\partial x_4} \tag{3}$$

whence we have for F_{24} an equation of the second degree

$$(1-\eta^2)\frac{\partial^2 F_{24}}{\partial r^2} - \frac{r}{a^2}\frac{\partial F_{24}}{\partial r} + \frac{\partial^2 F_{24}}{\partial x_4^2} = 0 \tag{4}$$

Its solution has the form

$$F_{24} = F\sin\vartheta e^{k(x_4 - iaz)} \tag{5}$$

where $k = \omega/c$, $z = \arcsin\frac{r}{a}$, and F is an arbitrary constant. It may be remarked that in place of $F\sin\vartheta$ in (5) there should stand a constant independent of r or x_4 and varying with ϑ. For the oscillator, however, this dependence reduces to the factor $\sin\vartheta$.

The expansion of $\arcsin\frac{r}{a}$ in a series gives

$$F_{24} = F\sin\vartheta e^{i\left[\omega t - kr\left(1 + \frac{1}{2\cdot 3}\frac{r^2}{a^2} + \cdots\right)\right]} \tag{6}$$

It may be seen from this that the velocity of propagation is less than c, if c is understood to be a constant entering into the metric in the form of $dx_4^2 = -c^2dt^2$ (see (2)). And it grows smaller, moreover, with increasing r, *i. e.* in the direction away from the source.

In the sequel, however, we shall use the solution in the general form (5).

The tensor of energy-impulse of an electromagnetic wave τ_i^k which in the general case is equal to

$$\tau_i^k = +F^{kl}F_{il} + \frac{1}{4}g_i^k F^{mn}F_{mn} \tag{7}$$

will here be equal to

$$\tau_1^1 = \frac{F_{24}^2}{r^2}; \quad \tau_4^4 = -\frac{F_{24}^2}{r^2}; \quad \tau_4^1 = \frac{iF_{24}^2\sqrt{1-\eta^2}}{r^2} \tag{8}$$

The rest of the components are equal to zero.

3. Gravitational Radiation in a Cylindrical World. When no electromagnetic wave is present, the gravitational equation

$$R_{0i}^k - \frac{1}{2}\delta_i^k R_0 = -\varkappa T_{0i}^k \tag{9}$$

has as its solution the metric of the cylindrical world *.

$$-ds^2 = e^{\lambda_0}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) + dx_4^2 \tag{10}$$

where $\lambda_0 = -\ln(1-\eta^2)$ (see (2)).

* We leave out the cosmological term in writing the equation (9). It may indeed be easily proved that while changing an equation of degree zero this term has no effect on an equation of the first degree, and so is of no importance to us.

In the presence of an electromagnetic wave the tensor of matter and energy T_i^k receives an addition in the form of the tensor of energy-impulse of this wave τ_i^k, with the result that the equations of gravitation will be

$$R_i^k - \frac{1}{2}\delta_i^k R = -\varkappa T_i^k \qquad (11)$$

where

$$T_i^k \equiv T_{0i}^k + \tau_i^k$$

The metric that satisfies this equation differs from (10) by small quantities characterizing the perturbation introduced into the metric by the electromagnetic wave. These perturbations depend on r, x_4 and ϑ. The metric that is a function of r, x_4 and ϑ can in the general case be written as follows

$$-ds^2 = e^{\lambda_1}dr^2 + r^2e^{\mu}(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) + e^{\nu}dx_4^2 + a_1 dr\,dx_4 + a_2 dr\,d\vartheta + a_3 d\vartheta\,dx_4 \qquad (12)$$

where $\lambda_1 = \lambda_0 + \lambda$; λ, μ, ν, a_1, a_2, a_3 are small quantities equal to zero in the absence of electromagnetic wave. We can subject r and x_4 to any infinitesimal tranformations without altering thereby the mertic of zero approximation (10). So let us choose them so as to have (2)

$$\mu = 0, \quad a_1 = 0 \qquad (13)$$

Taking (13) into account, substitute the metric (12) into (11) and develop this equation into a power series of small quantities. The terms of zero approximation not containing these quantities offset one another by virtue of (9), and the linear terms with τ_i^k in the right-hand sides give a system of linear differential equations for determining the unknown functions λ, ν, a_2, a_4.

Omitting very cumbersome and long computations, we shall write here down the solutions of these equations at once

$$\left.\begin{aligned} a_2 = a_4 = 0; \quad \lambda &= \frac{\varkappa F^2 \sin^2\vartheta \sin 2(\omega t - kaz)}{2kr\sqrt{1-\eta^2}} \\ \nu &= \lambda + \frac{\varkappa F^2 \sin^2\vartheta \cos 2(\omega t - kaz)}{2k^2r^2} \end{aligned}\right\} \qquad (14)$$

$1/k^2r^2$ are neglected throughout being small in comparison with unity. It may be seen that in the plane world, too ($a = \infty$), λ and ν are different from zero.

We shall now compute the energy lost by the electromagnetic wave on gravitational radiation. For this we can use the well-known expression for the pseudotensor of impulse-energy of the gravitational wave.

We find from it that the loss of energy by an electromagnetic wave on gravitational radiation in a spherical layer between r and $r+dr$ is

$$A = \frac{8\pi i \varkappa F^4\, dr}{15(1-\eta^2)k^2r^3}(1 - 5\eta^2) \qquad (15)$$

This expression has been obtained upon neglecting $\frac{1}{k^2r^2}$ in comparison with unity. Taking these terms into account gives additional small terms containing $\frac{1}{k^4r^5}$, and so on.

If we were to develop the expression (15) in a series of powers of $\eta^2 = (r/a)^2$, we should obtain summands with the following dependence on r: dr/r^3, dr/r, rdr, ..., in other words, there would be no term among them proportional to dr. This means that there is no analogue to the Cherenkov effect in the cylindrical world, for this effect implies the presence of a radiation that is invariable with reference to a unit path (in our case a unit thickness of the spherical layer). We may well have expected this result, since according to the solutions (5) and (14) the velocities of electromagnetic and gravitational waves fail here to coincide.

The radiation given by formula (15) is so insignificant as to defy detection by experiment. To make this clear, we can compare the losses by gravitational

radiation along the entire path of the wave up to the distance R with the total flux of electromagnetic energy through a sphere of this radius. This flow, as follows from (8), after integrating over angles and multiplying by R^2 is equal to F^2 in order of magnitude. The losses due to gravitational radiation are obtained by integrating A with respect to r.

Assuming, in order to make an approximate estimate, that the expression (15) for A is valid, say, for $kr \gg 1$, we obtain after integration from $r = 1/k$ to R the following expression

$$\frac{4\pi\varkappa F^4}{15}\left(1-\frac{1}{k^2R^2}\right)$$

(for simplicity we have put $\eta = 0$, *i. e.* $a = \infty$).

Neglecting the second term in parentheses we obtain for great distances from the source

$$\frac{4\pi\varkappa F^4}{15} \qquad (16)$$

The ratio of this value to the total flux of electromagnetic energy is a magnitude of the same order as $\varkappa F^4$. As F^2 is small and the order of $\varkappa$ is 10^{-48}, it will be obvious that the decrease in the flux of electromagnetic energy due to gravitational radiation is as small as to be inobservable.

4. Generalization to the Spherically Isotropic World. The metric of any spherical isotropic world can be represented as the metric of a cylindrical world (2), except that the radius of a three-dimensional cross-section of the world a will be a function of x_4. The solutions for λ and ν may be written as a power series of $\eta^2 = (r/a)^2$. Confining ourselves to the first term of this series we obtain

$$\lambda = f(1 + B\eta^2); \quad \nu = \lambda + g(1 + D\eta^2) \qquad (17)$$

where f and g are the solutions for the plane world. They can be obtained from (14) putting $a = \infty$. The constants B and D are not known, but they are certainly magnitudes of order 1. Using these solutions for λ and ν and remembering that a is a function of x_4, we get

$$\overline{A} = \frac{\varkappa F^4 \sin^2\vartheta}{k^2r^2}\left[\frac{i}{2r}(1-4\eta^2) + \frac{1}{a}\frac{da}{dx_4}(M+N\eta^2)\right] \qquad (18)$$

where M and N are magnitudes of order 1. For $a = \text{const}$ (cylindrical world) we obtain the same result as before.

Note that here the terms in A are independent of r, so we have here a constant radiation per a unit path, in other words, a phenomenon analogous to the Cherenkov effect. However, these terms do not stand out against the others and are even small compared to them.

Thus, in the general case the effect has the same order of magnitude as in the cylindrical world, *i. e.* defies observation. For example, it can in no way account for the red displacement in the spectra of distant nebulae.

It also appears that a spherical electromagnetic wave, unlike a plane wave, propagates without losing any appreciable portion of its energy on gravitational radiation.

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Received
20.II.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

PHYSIQUE APPLIQUÉE

SUR LE PROCESSUS D'ÉTABLISSEMENT DE SYSTÈMES D'OSCILLATION A UN DEGRÉ DE LIBERTÉ

Par V. V. KASAKEVITCH

(Présenté par N. D. Papalexi, de l'Académie, le 24. VII. 1945)

La Note présente est consacré à une méthode de recherche du processus d'établissement de systèmes oscillatoires quand les perturbations extérieures font défaut.

1. Considérons l'équation

$$\ddot{s} + f(s, \dot{s}) = 0 \qquad (1)$$

Soit $\dot{s} = v$; écrivons-la sous la forme

$$\frac{ds}{dt} = v, \quad \frac{dv}{dt} = -f(s, v) \qquad (1a)$$

Supposons que la résolution du système $v = 0$, $f(s, v) = 0$ donne les positions d'équilibre s_i, $v_i = 0$ $(i = 1, 2, \ldots, k)$. Il est facile de montrer que le mouvement autour de la position d'équilibre quelconque s_i, v peut être défini par quatre systèmes dynamiques indépendants

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -f(|x|, -|y|)\operatorname{sgn} x \qquad (2a)$$

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = f(-|x|, -|y|)\operatorname{sgn} x \qquad (2b)$$

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = f(-|x|, |y|)\operatorname{sgn} x \qquad (2c)$$

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -f(|x|, |y|)\operatorname{sgn} x \qquad (2d)$$

$$x = s = s_i, \quad y = v$$

dont les courbes dans le plan de phase x, y sont symétriques par rapport aux axes de coordonnées*.

Les systèmes (2a), (2b), (2c) et (2d) décrivent le mouvement du système initial respectivement dans le quatrième, troisième, deuxième et le premier quadrant du plan de phase x, y (1a). Nous allons appeler les équations de la forme (2) quasi-conservatives à cause de la symétrie des courbes intégrales**. Par suite de cette symétrie il est suffisant de n'avoir la solution du système (2) que pour l'un des quadrants du plan de phase.

Supposons que les équations des courbes intégrales des systèmes (2) soient trouvées. Soit, pour les valeurs initiales $t = t_0$, $x = a$, $y = 0$

$$y_i = y_i(x, a, 0) \quad (i = 1, 2, 3, 4) \qquad (3)$$

* Un des cas particuliers d'un tel remplacement a été employé par l'auteur en [1].

** Le cas du centre relié à la symétrie par rapport à une droite passant par l'origine des coordonnées, a été examiné dans les travaux [2-4].

Si l'on prend dans (3) $x=0$, on obtient de certaines fonctions de valeurs initiales (des amplitudes) que nous désignerons fonctions déterminantes

$$v_i(a) = y_i(0, a, 0) \tag{4}$$

Considérons $v_i(a)$ $(i=1, 2, 3, 4)$ dans les quatrièmes quadrants des plans de phase. Alors, si la fonction $f(x, y)$ est continue et satisfait aux conditions de Lipshitz, $v_i(a)$ sont des fonctions univoques, qui ne sont pas définies pour toutes les valeurs de a et décroissent d'une telle manière que pour chaque couple de nombres a_1 et a_2, pour lesquels v_i existent et tels que $a_1 < a_2$, l'on a $v_i(a_1) \leq v_i(a_2)$.

Dans le cas où une amplitude initiale $a_1 > 0$ est donnée, sur laquelle existent v_i, la détermination du processus d'établissement se réduit à la recherche successive de $a_2, a_3, \ldots$ à partir des équations

$$\begin{aligned} &v_1(a_1) = v_2(a_2),\ a_2 = a_3,\ v_3(a_3) = v_4(a_4),\ a_4 = a_5 \\ &v_1(a_5) = v_2(a_6),\ a_6 = a_7,\ v_3(a_7) = v_4(a_8),\ a_8 = a_9 \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \tag{5}$$

Si dans le système (1a) ont lieu des mouvements périodiques stables (équilibre compris) enveloppant un point particulier $x=y=0$, et si la valeur initiale a_1 est prise dans le domaine d'attraction d'un de ces mouvements, l'amplitude stationnaire sera la limite d'une succession d'amplitudes a_2, $a_3, \ldots, a_{2n}, a_{2n+1}, \ldots$ pour $n \to \infty$. Le nombre de couples des racines réelles du système d'équations

$$v_1(\alpha) = v_2(\beta), \quad v_3(\beta) = v_4(\alpha) \tag{6}$$

et leurs valeurs déterminent le nombre et les valeurs des amplitudes stationnaires.

Soient α^*, β^* un couple de racines réelles de (6). L'amplitude stationnaire est asymptotiquement stable, si l'inégalité

$$\left| \frac{dv_1(\alpha^*)}{d\alpha} \frac{dv_3(\beta^*)}{d\beta} \right| < \left| \frac{dv_2(\beta^*)}{d\beta} \frac{dv_4(\alpha^*)}{d\alpha} \right| \tag{7}$$

est satisfaite.

Si le sens de l'inégalité est opposé, l'amplitude stationnaire est instable. Le cas de l'égalité est douteux.

2. La connaissance des fonctions déterminantes permet de trouver le processus d'établissement et les solutions périodiques des systèmes discontinus quelconques, satisfaisant dans chaque quadrant au théorème de Cauchy*.

Tout système dynamique de la forme (1a) peut être considéré comme un certain mécanisme forçant un point mobile dans chaque quart d'une oscillation à se déplacer conformément à une équation différentielle définie (2a).

Pour (1a) le nombre m des équations différentielles, pour lequel l'ordre de leur replacement devient périodique, peut prendre des valeurs suivantes: 1) $m=1$ (système conservatif et non-conservatif symétriques par rapport aux axes des coordonnées $v_1 = v_2 = v_3 = v_4$); 2) $m=2$ (système non-conservatif symétrique par rapport à l'origine $v_1 = v_3$, $v_2 = v_4$); 3) $m=4$ (systèmes conservatif et non-conservatif symétriques par rapport à l'axe des abscisses $v_2 = v_3$; système non-conservatif symétrique par rapport à l'axe des abscisses $v_2 = v_3$; système non-conservatif symétrique par rapport à l'axe des ordonnées $v_1 = v_2$; $v_3 = v_4$; système non-symétrique et non conservatif, tous les v_i sont distincts).

Tout de même on peut imaginer un système discontinu, dans lequel m prend également d'autres valeurs. Il faut alors distinguer deux cas: a) $m=2n$, b) $m=2n-1$ $(n=1, 2, \ldots)$.

* De même, peuvent être facilement étudiés de tels systèmes discontinus, comme un régulateur électromagnétique de tension de Tirill, comme «un système technique d'équations» décrivant, d'après Levental, le processus d'établissement dans le système de l'automate «VIC», *etc.*

Pour $m = 2n$ le nombre et les amplitudes des solutions périodiques seront définis par le nombre et par les valeurs des racines réelles d'un système de n équations aux n inconnues $\alpha_1, \alpha_2, \ldots, \alpha_n$:

$$v_1(\alpha_1) = v_2(\alpha_2),\ v_3(\alpha_2) = v_4(\alpha_3),\ \ldots,\ v_{2n-1}(\alpha_n) = v_{2n}(\alpha_1) \qquad (8)$$

Le nombre des amplitudes distinctes de chaque mouvement périodique est $\leqslant m : 2$.

Soient $\alpha_1^*, \ldots, \alpha_n^*$ les amplitudes du mouvement périodique. Le mouvement périodique est stable, si l'inégalité

$$\left| \prod_{i=1}^{n} \frac{dv_{2i}(\alpha_i^*)}{d\alpha_i} \right| < \left| \frac{dv_n(\alpha_1^*)}{d\alpha_1} \prod_{i=1}^{n} \frac{dv_{2i}(\alpha_{i+1}^*)}{d\alpha_{i+1}} \right| \qquad (9)$$

est satisfaite. Si le signe de l'inégalité est opposé, le mouvement est instable. Le cas de l'égalité est douteux.

Pour $m = 2n - 1$ l'on a $2n - 1$ équations à $2n - 1$ inconnues $\alpha_1^*, \alpha_2^*, \ldots, \alpha_{2n-1}^*$

$$\begin{gathered} v_1(\alpha_1) = v_2(\alpha_2),\ v_3(\alpha_2) = v_4(\alpha_3),\ \ldots,\ v_{2n-1}(\alpha_n) = v_1(\alpha_{n+1}) \\ v_2(\alpha_{n+1}) = v_3(\alpha_{n+2}),\ \ldots,\ v_{2n-2}(\alpha_{2n-1}) = v_{2n-1}(\alpha_1) \end{gathered} \qquad (10)$$

dont le nombre et les valeurs des racines définissent les solutions périodiques. Le nombre de diverses amplitudes de chaque mouvement périodique est $\leqslant m$.

Soient $\alpha_1^*, \alpha_2^*, \ldots, \alpha_{2n-1}$ les amplitudes du mouvement périodique. Le mouvement périodique est stable si l'inégalité

$$\left| \prod_{i=1}^{n} \frac{dv_{2i-1}(\alpha_i^*)}{d\alpha_i} \prod_{i=1}^{n-1} \frac{dv_{2i}(\alpha_{n+1}^*)}{d\alpha_{n+i}} \right| < \left| \frac{dv_{2n-1}(\alpha_1^*)}{d\alpha_1} \prod_{i=1}^{n-1} \frac{dv_{2i}(\alpha_{i+1}^*)}{d\alpha_{i+1}} \prod_{i=1}^{n-1} \frac{dv_{2i-1}(\alpha_{n+i}^*)}{d\alpha_{n+i}} \right| \qquad (11)$$

est satisfaite.

Si le signe de l'inégalité est opposé, le mouvement est instable. Le cas d'égalité est douteux.

3. Il est commode de déterminer le processus d'établissement par la méthode graphique, en traçant dans le plan $v - a$ les courbes des fonctions déterminantes. Si l'amplitude initiale a_1 (ou bien a_1') est choisie, les amplitudes suivantes $a_2, \ldots$ (ou bien $a_2', \ldots$) se trouveront d'après (5), au moyen de la construction représenté sur la fig. 1, où est exposé le processus d'établissement pour l'équation

$$x - 0.1\pi\,(1 - 2x + x^2)\,x + x = 0$$

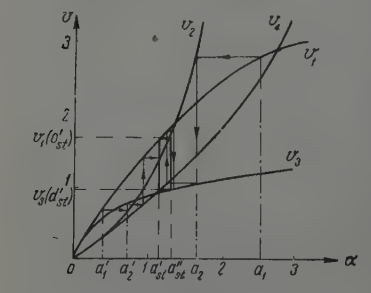

Fig. 1

L'asymétrie du système s'y révèle bien nettement par le fait que le cycle limite se représente par un rectangle déterminant les amplitudes du cycle limite a'_{st} et a''_{st}, de même que les vitesses $v_1(a'_{st})$ et $v_3(a'_{st})$. La construction montre l'instabilité de l'équilibre et la stabilité du cycle limite.

Pour les systèmes symétriques, étant donné que l'on a

$$v_1(a) \equiv v_3(a), \quad v_2(a) \equiv v_4(a)$$

le rectangle déterminant le cycle limite se transforme en point. Le procédé d'itération (5) sur le plan $v - a$ peut être inscrit schématiquement de la façon suivante

$$\begin{array}{ccccc} v_1(a_1) & \longrightarrow & v_2(a_2) & \quad v_1(a_5) \longrightarrow v_2(a_6) & \\ & & \downarrow & \uparrow & \downarrow \\ & & v_3(a_3) & \longrightarrow v_4(a_4) & v_3(a_7) \longrightarrow etc. \end{array} \qquad (12)$$

et peut nous amener. a_1 étant convenablement choisi, aux amplitudes des mouvements périodiques stables (équilibre compris). Les amplitudes des mouvements périodiques instables peuvent être trouvées par un procédé inverse à celui, qui a été décrit plus haut:

$$\begin{array}{ccccc} v_1(a_1) & & v_2(a_4) & \longrightarrow & v_1(a_5) \\ \downarrow & & \uparrow & & \downarrow \\ v_4(a_2) & \longrightarrow & v_3(a_3) & & v_4(a_6) \longrightarrow etc. \end{array} \tag{12a}$$

4. Considérons quelques exemples.

1) Les oscillations harmoniques amorties $\ddot{x} + 2b\dot{x} + k^2x = 0$. Les fonctions déterminantes ont dans ce cas la forme

$$v_1(a) = v_3(a) = -ake^{-\frac{b}{k_1}\left[\frac{\pi}{2} + \operatorname{arctg}\frac{b}{k_1}\right]}$$

$$v_2(a) = v_4(a) = -ake^{\frac{b}{k_1}\left[\frac{\pi}{2} - \operatorname{arctg}\frac{b}{k_1}\right]}$$

$$k_1 = \sqrt{k^2 - b^2}$$

Deux amplitudes successives a_n et a_{n+1} se déterminent à partir de la condition $v_1(a_n) = v_2(a_{n+1})$, d'où

$$\frac{a_n}{a_{n+1}} = \frac{e^{\frac{b}{k_1}\left[\frac{\pi}{2} - \operatorname{arctg}\frac{b}{k_1}\right]}}{e^{-\frac{b}{k_1}\left[\frac{\pi}{2} + \operatorname{arctg}\frac{b}{k_1}\right]}} = e^{\frac{\pi b}{k_1}} = \text{const}$$

Le processus d'établissement dans le plan $v - a$ est présenté sur la fig. 2. Pour $b = 0$ les fonctions déterminantes coïncident et sont représentées par la droite $v = ak$. Lorsque b croît, $v_1(a)$ et $v_2(a)$ commencent à tourner autour de l'origine des coordonnées en s'éloignant l'une de l'autre.

Pour $b = k$ la droite $v_1(a)$ coïncide avec l'axe des abscisses. La construction montre nettement l'influence de friction linéaire (la diminution des amplitudes en progression géométrique).

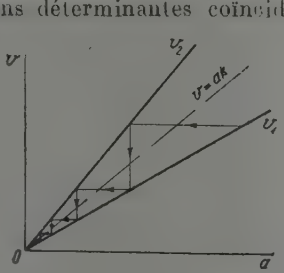

Fig. 2

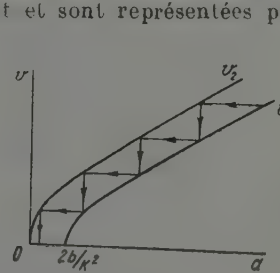

Fig. 3

2) La friction de Coulomb $\ddot{x} + k^2x = f(\dot{x})$, où $f(\dot{x}) = -B \operatorname{sgn} \dot{x}$ pour $\dot{x} \gtrless 0$; $-B \leqslant f(\dot{x}) \leqslant B$ pour $\dot{x} = 0$.

Les fonctions déterminantes sont de la forme

$$v_1(a) = v_3(a) = -\sqrt{a^2k^2 - 2Bk}, \quad \frac{2B}{k^2} \leqslant a < +\infty$$

$$v_2(a) = v_4(a) = -\sqrt{a^2k^2 + 2Bk}, \quad 0 \leqslant a < +\infty$$

Étant donné a_1, l'on a d'après la condition $v_1(a_1) = v_2(a_2)$

$$a_2 = a_1 - \frac{2B}{k^2} \quad etc., \quad a_n = a_1 - \frac{2nB}{k^2}$$

Le mouvement a lieu autant que l'on a $a > B/k^2$.

Sur la fig. 3 est représenté le processus d'établissement dans le plan $v - a$. On y voit bien la conséquence de friction de Coulomb (diminution des amplitudes en progression arithmétique).

Manuscrit reçu
le 24. VII. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

TECHNICAL PHYSICS

APLANATIC MENISCUS TELEOBJECTIVES

By D. MAKSUTOV

(Communicated by G. A. Shajn, Member of the Academy, 23. I. 1945)

I have considered in detail in my previous paper(1) the simplest aplanatic system «meniscus—concave mirror»; there I have touched but slightly upon the possibility of constructing compound aplanatic meniscus systems of the type of teleobjectives. Of these the greatest practical value should be ascribed to the system which was designated by me as the «meniscus Cassegrain» (Fig. 1). The following are the symbols: A, the «achromatic» meniscus; B, the primary concave spherical mirror; C, the secondary convex spherical mirror; R_1, R_2, R_3, R_4, the four radii of curvature of the optical surfaces; d_1, d_2, d_3, the thickness of the meniscus and the two air intervals; S', the ultimate segment.

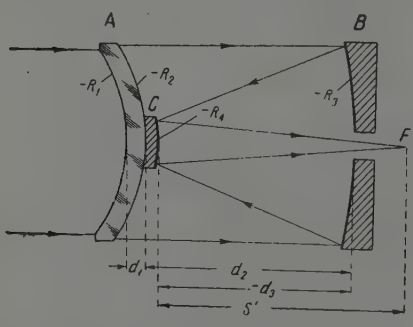

Fig. 1

For the mass production of inexpensive instruments the systems are of interest which satisfy the following condition

$$R_4 = R_2, \quad d_3 = -d_2 \qquad (1)$$

since here the secondary mirror C is simply an aluminized central area of the inner surface of the meniscus.

Such systems, however, when rationally designed, cannot be made in the form of aplanatic systems: their spherical aberration is corrected, but coma is several times greater than in an equivalent Cassegrain system of mirrors or, which is the same, in a single equivalent parabolic mirror.

Besides, coma has here the opposite sign.

Let us express the coma of the system by the generally adopted quantity

$$\frac{\Delta f'_y}{f'} = \frac{\Delta S'_y}{S' - x'} \qquad (2)$$

We shall compare it with the coma of the equivalent parabolic mirror, *i. e.* with a mirror having the same value of the zone y and the same focal length f' with the meniscus system under comparison. The coma of the parabolic mirror is known to be expressed by the quantity

$$\frac{\Delta' f'_y}{f'} = 0{,}0625 \left(\frac{2y}{f'}\right)^2 \qquad (3)$$

As an example may serve the system «meniscus Cassegrain» of the aperture $D=500$ mm, which satisfies the condition (1). We shall choose the fol-

lowing design which suits the case well:

System I

$$\begin{aligned} R_1 &= -569.42 & d_1 &= 52.19\,(K-8) \\ R_2 &= -600.21 & d_2 &= 1010.49 \\ R_3 &= -2451.05 & d_3 &= -d_2 \\ R_4 &= R_2 \end{aligned} \qquad (4)$$

$$D : f' = 1 : 12.72$$
$$S'_0 = 1300$$

The wave aberrations (h_y) of such a system * are shown in Fig. 2, curve *I*, which shows that the design elements were properly chosen to correct spheri-

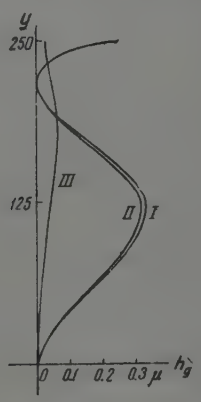

Fig. 2

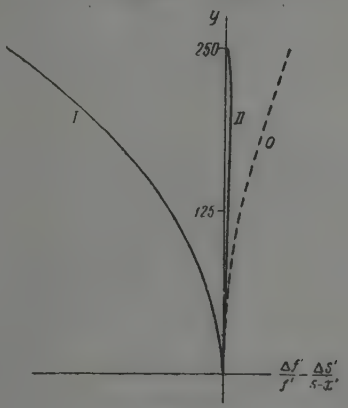

Fig. 3

cal aberration with the given meniscus. Fig. 3, curve *I*, shows the coma of this system; by comparing curves *I* with the dotted curve *O* showing the coma of the equivalent parabolic mirror, we find that coma is in our system three times as great as in the equivalent paraboloid and has an opposite sign («inner coma»).

To accomplish an aplanatic system we are forced to give up condition (1). Without changing meniscus we may pass from system I to an aplanatic system II; the corresponding elements of design should be as follows:

System II

$$\begin{aligned} R_1 &= -569.42 & d_1 &= 52.19\,(K-8) \\ R_2 &= -600.21 & d_2 &= 841.13 \\ R_3 &= -2311.69 & d_3 &= -843.63 \\ R_4 &= -965.55 \end{aligned} \qquad (5)$$

$$D : f' = 1 : 7.512$$
$$S'_0 = 1126$$

* The calculations and the curve relate to the blue ray *F*.

Curve *II*, Fig. 2, shows that in spite a substantial modification in the design elements of the system and in its focal ratio, there was practically no change in the wave aberrations. That is, in an adequately designed system wave aberrations are mainly determined by the parameters of the meniscus.

Curve *II* of the Fig. 3 shows that the best correction of the coma is attained in System *II* and that the residual coma is negligible as compared with the coma of the equivalent paraboloid (curve *O*). System *II* can, therefore, be called aplanatic or, to be more precise; it conforms in the best manner to the requirements of i s o p l a n a t i o n.

It may be seen that when we pass from a non-aplanatic system I to the aplanatic system II, the radius R_4 should be made nearly 1.6 longer. That the radius

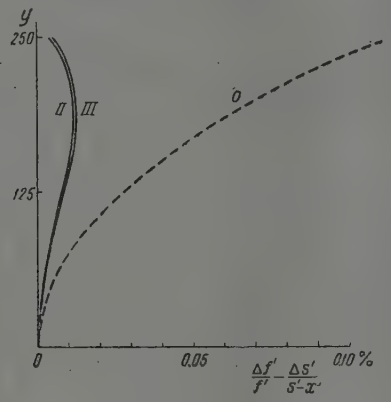

Fig. 4

should be lengthened, not shortened. is a favourable fluke, since it makes it possible to grind the reflecting surface of the radius R_4 on the inner surface of the meniscus; otherwise we would be faced with the necessity to make a separate secondary mirror *C* (Fig. 1) and to fasten it somehow to the meniscus, which is improfitable from the designer's point of view. The intervals d_2 and d_3 are selected in such a way as to give the secondary mirror, which is ground on the meniscus, a desirable diameter.

The residual spherical aberration (curve *II*, Fig. 2) may be removed or substantially reduced by figuring one of the four optical surfaces. For many reasons it is profitable to make the figuring on the f i r s t s u r f a c e o f t h e m e n i s c u s.

The longitudinal spherical aberration ΔS_y^0 measured from the most suitably chosed focusing plane may be represented as follows:

$$\Delta S_y^0 = -\Delta + \alpha\left(\frac{y}{H}\right)^2 + \beta\left(\frac{y}{H}\right)^4 \qquad (6)$$

We shall compute for our case the value of defocusing Δ with reference to Gaussian plane and the coefficients α and β which are

$$\Delta = -1.44 \text{ mm}, \quad \alpha = -7.92 \text{ mm}, \quad \beta = +7.77 \text{ mm} \qquad (7)$$

Now the equation of the deformed first surface of the meniscus leading in the first approximation to the elimination of the residual spherical aberration will assume the following form:

$$x = R_1\left(1 - \sqrt{1 - \frac{y^2}{R_1^2}}\right) + \frac{1}{4(n-1)}\left(\frac{D}{f'}\right)^2\left[-\frac{\Delta}{2}\left(\frac{y}{H}\right)^2 + \frac{\alpha}{4}\left(\frac{y}{H}\right)^4 + \frac{\beta}{6}\left(\frac{y}{H}\right)^6\right] \qquad (8)$$

The trigonometric calculation of such a system III possessing a figured first surface of the meniscus demonstrates a satisfactory correction of the residual spherical aberration (curve *III*, Fig. 2). It should be noted that figuring the first surface of the meniscus (which is the nearest to the entrance pupil of the instrument) had practically no influence upon the correction of the coma attained earlier. Fig. 4 shows the curves of the residual coma of the unfigured (curve *II*) and of the figured (curve *III*) aplanatic systems, the curve *O* showing, as usual, the coma of the equivalent parabolic mirror.

It turns out that such a figuring had practically no influence neither upon the ultimate segment, nor upon the focal ratio, astigmatism or curvature of the field. Without presenting the curves of the astigmatism, we shall note that the astigmatism of the given system is negligibly small. Thus, with an angular field $W=1°$ (diameter of the field $2W=2°$) the astigmatic difference will be only 0.21 mm. The radius of curvature of the field of vision approximates 950 mm and, therefore, the curvature of the image may be easily corrected by the application of the Piazzi-Smith lens.

The following procedure may therefore be recommended for manufacturing figured aplanatic meniscus of the Cassegrainian type:

1. A system is computed with spherical surfaces which is similar to system II where spherical aberration is corrected in the best possible way and which conforms the requirements of isoplanatism.

2. The optician is charged with the task to figure the first surface of the meniscus until the residual spherical aberration disappears or is considerably reduced; this operation should be checked by the shadow test in the autocollimation scheme.

3. All the aberrations computed before, except for the spherical aberration which has been eli nated, will suffer no noticeable changes.

Lack of space prevents from presenting the analogous computations and conclusions for the meniscus teleobjectives of the Gregorian type.

In concluding I wish to express my deep appreciation of the assistance lent by V. V. Danilova who successfully made a great many trigonometric computations necessary for the present investigation.

Received
23.I.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

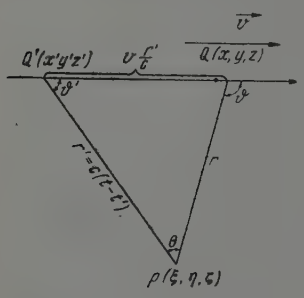
TECHNICAL PHYSICS

THE SOUND FIELD OF MOVING SOUND SOURCES

By M. F. SHIROKOV

(Communicated by B. A. Vvedensky, Member of the Academy, 30. I. 1945)

The question with which this paper is concerned does not appear to have been ever discussed consistently on theoretical lines. It is an important question, however, as it bears on many practical problems, such as the calculation of the sound field of an airplane propeller in flight, of sound radiation into the atmosphere from a source of sound, when influenced by a wind, and so on.



The problem in such cases is reduced to finding a new solution of the differential equations of acoustics with sources moving in the medium. It is this general theoretical treatment of the subject that will be attempted here.

The equation of acoustics with sources that are not chosen artificially but are rather involved in the equations of hydrodynamics, can only be obtained from the latter in the form indicated by Lamb (1):

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p=\operatorname{div}\vec{K} \qquad (1)$$

where p is sound pressure at a certain point in a gaseous medium, c is velocity of the sound, and $\vec{K}$ is a force per unit volume of a periodical character responsible for the acoustic effect in the medium.

The field of force due to sources of frequency ω moving with a constant velocity $\vec{v}$ in the direction of the x-axis can be written as follows:

$$\vec{K}=\vec{K}_0(x-vt,\ y,\ z)\,e^{i\omega t} \qquad (2)$$

Analogous expressions can be given to the components of the force $\vec{K}$ along the coordinate axes X, Y and Z.

Let p be the sound pressure at an instant t at the point of observation $P(\xi, \eta, \zeta)$ and

$$t'=t-\frac{r'}{c} \qquad (3)$$

$$r'=\sqrt{(x'-\xi)^2+(y'-\eta)^2+(z'-\zeta)^2} \qquad (4)$$

$t-t'=r'/c$ is the time the sound impulse takes to travel the distance from some arbitrary point in space Q' (x', y', z') to the observation point P (see figure).

During the interval $t-t'$ some arbitrary value of the amplitude of the field

of force $\vec{K}_0$ will be displaced from point $Q'(x', y', z')$ to point $Q(x, y, z)$, the coordinates of these points being connected by the relations

$$x' = x - \frac{vr'}{c}, \quad y' = y, \quad z' = z \tag{5}$$

It may easily be found from the figure that

$$\left.\begin{aligned} r' &= r\,\frac{\sqrt{1-\left(\frac{v}{c}\sin\vartheta\right)^2}+\frac{v}{c}\cos\vartheta}{1-\frac{v^2}{c^2}} \\ \cos\vartheta' &= \frac{v}{c}+\frac{\left(1-\frac{v^2}{c^2}\right)\cos\vartheta}{\sqrt{1-\left(\frac{v}{c}\sin\vartheta\right)^2}+\frac{v}{c}\cos\vartheta} \\ 0 &= \vartheta - \vartheta' \end{aligned}\right| \tag{6}$$

By means of (4) and (5) we obtain, making use of the theory of delayed potentials, the solution of (1) in the form

$$p = \frac{e^{i\omega t}}{4\pi}\int_{\Omega}(\vec{K}_0 \operatorname{grad} f)\,d\Omega \tag{7}$$

where $d\Omega = dx\,dy\,dz$ and $\vec{K}_0$ and f are functions of the coordinates of the field of force at the instance t. By virtue of (5) and (6) we have here

$$\frac{e^{-\frac{i\omega r'}{c}}}{r'\left(1-\frac{v}{c}\cos\vartheta'\right)} = f(x, y, z) \tag{8}$$

Let us apply (7) to a point source with a field of force $\vec{K}$ concentrated in the region Ω whose dimensions are small in comparison with r' and with the wave-length $\lambda = 2\pi c/\omega$ radiated by the source.

Suppose, moreover, that the point of observation is so faraway from the sources that the variation of $r'^{-1}\left(1-\frac{v}{c}\cos\vartheta'\right)^{-1}$ inside Ω may be neglected. Then (7) will yield the following approximate relation where ϑ'_x, ϑ'_y and ϑ'_z denote the angles formed by the vector $\overrightarrow{Q'P} = r'$ with the coordinate axes x, y and z:

$$p = \frac{B'}{r'\left(1-\frac{v}{c}\cos\vartheta'_x\right)}\,e^{-i\omega\left(t-\frac{r'}{c}\right)} \tag{9}$$

Here

$$B' = \int_{\Omega}\frac{i\omega}{4\pi c}\left(\frac{X_0\cos\vartheta'_x}{1-\frac{v}{c}\cos\vartheta'_x}+Y_0\cos\vartheta'_y+Z_0\cos\vartheta'_z\right)d\Omega \tag{10}$$

X_0, Y_0, Z_0 being the components of $\vec{K}_0$. This formula resembles the well-known relations for the Liénard and Wiechert potentials established for a moving electron (²). There is, however, an essential difference. The quantity B' which plays the rôle of the electron charge varies with the state of motion of the source, and this variation can be rather considerable. The reason lies in the fact that the sound sources can be obtained from the equations of hydrodynamics only in the form of divergence of a certain force $\vec{K}$ per unit volume. It is mainly owing to this circumstance that the amplitude of a point source is connected with the quantity $\varepsilon'c$ by a more complicated relation than in the case of Liénard-Wiechert potentials.

It follows from (9) that the phase of a point-source as perceived at the observation point P is

$$\alpha = \omega\left(t - \frac{r'}{c}\right) \qquad (11)$$

since its relation to t involves r' as well. Instead of the frequency ω with which oscillates the field of force $\vec{K}$, the observer at point P will record another frequency which, according to (2) and (5), will be

$$\omega_1 = \frac{d\alpha}{dt} = \frac{\omega}{1 - \frac{v}{c}\cos\vartheta'} \qquad (12)$$

The relation (12) expresses the Doppler effect which is here obtained from the solution of the differential equation of acoustics (1) instead of from the elementary and purely kinematic conceptions on the propagation of waves from a moving source. This formula is an approximation because it is derived from the approximate relation (9), and also because ω_1 cannot be considered to be the frequency unless the course of variation of α during the time $T_1 = 2\pi/\omega_1$ is reproduced satisfactorily by the development of (11) into a Taylor series in which all terms, following that containing $d\alpha/dt$ are discarded.

If the acoustic field be supposed to be described by the velocity potential φ, then, denoting the average density of the medium by ρ, we shall have, according to (9)

$$\varphi = -\frac{iB'}{r'\rho\omega} e^{i\omega\left(t - \frac{r'}{c}\right)} \qquad (13)$$

On the other hand, it may be deduced from the equations of hydrodynamics that the flux of acoustic energy is

$$\vec{q} = p\vec{u} \qquad (14)$$

where $\vec{u}$ is the vibration velocity of the particles of the medium.

As $\vec{u} = -\operatorname{grad}\varphi$, the vector $\vec{q}$ is orthogonal to the surface $\varphi = \text{const}$. At a point of observation P (see figure) that is far away from the source, the vector $\vec{q}$, in accordance with (13), will be directed along $\overrightarrow{Q'P}$. Now the direction of the sound ray coincides with the direction of $\vec{q}$. Therefore the ear of the observer will locate the source in the direction $\overrightarrow{PQ'}$, while his eye will see it in the direction $\overrightarrow{PQ}$ (see figure). The angle ϑ is therefore the aberration angle. It can be computed from the relations (6). As in the case of Doppler effect, we have here an aberration obtained as a consequence from the solution of the differential equation of acoustics.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

PHYSICAL CHEMISTRY

A NEW EXPRESS METHOD OF PHOSPHATING STEEL PARTS

By G. V. AKIMOV, Corresponding Member of the Academy, and A. A. ULYANOV

Phosphating is one of the most widespread ways of protecting mass produced steel parts from atmospheric corrosion. The method customarily employed is complete phosphating, but the time of treatment is indefinite, frequently being much too long (as much as two and even more hours)*, and herein lies an essential shortcoming ([1–4]).

Even when the process of complete phosphating is carried out under normal conditions, it is markedly dependent upon a number of factors ([2,4,5]): the composition and preliminary treatment of the surface of the metal ([3,6]), the concentration, acidity and temperature of the bath [([5,7]) and ([8]), page 345], and the presence in the latter of ions SO''_4, Cl' or other impurities.

The express processes widely used abroad (Parker, Bonder, Atrament) (([8]), page 362 and([9–13])) are kept in secret and only their most general features are known. The speeding up of the process attained herewith gives rise to a number of new shortcomings: an abrupt drop in quality of the coating and extreme difficulties in regulating the course of the process [([6]) and ([8]), page 345].

All this made phosphating frequently hard to carry out under wartime production conditions, and an array of new problems forced themselves upon the attention of those concerned with this branch of industry. These were

1) to speed up the process without deteriorating the quality of the coating;
2) to make it possible to keep to a strict time schedule;
3) to reduce spoilage due to faulty coating to a minimum, and
4) to effect a saving in the main constituents of the phosphating reagents (Russian trade mark MAGEF).

These requirements are taken into account in the new express (incomplete) method of phosphating as proposed by the author**. In tackling this problem we were guided by the following considerations.

1. The stability of the protective film is not in general directly dependent upon its thickness. However, films of intermediate thickness may be said to have the best protective properties ([13]). They are more uniform, and free from the large internal stresses characteristic of thick films. At the same time they are a sufficiently strong barrier to the diffusion of the corrosive medium into the metal.

2. The main protective rôle is played by the comparatively thin layer of the film immediately adjacent to the metal.

3. When the phosphating process is protracted, film growth is not uniform with respect to either thickness or grain size, resulting in a heterogeneous structure which offers a poor resistance to corrosion at the boundaries between its unlike parts.

* Until the end of «boiling», *i. e.* of the evolution of hydrogen, and additional retention of the parts in the bath (10—15 min.) for control purposes.

** The process is interrupted before evolution of hydrogen runs to completion.

4. Cessation of the evolution of hydrogen, which usually serves as an indicant of the end of film growth (the phenomenon of self-repairing), is no proof of the resistance of the film to other corrosive media.

5. Should the protection provided by the thinner and more porous films formed by incomplete phosphating prove insufficient, as the case may be, we have such effective means of bolstering their protective properties as impregnation with protective inhibitors (chromates, for example) and organic substances (lubricants)*.

6. «Accelerated» films should not show any striking difference either from those formed by complete phosphatization as regards adherence to the metal, uniformity and mechanical properties.

7. Acceleration of the phosphating process should appreciably cut down consumption of phosphating reagents and film spoilage (as regards ex-

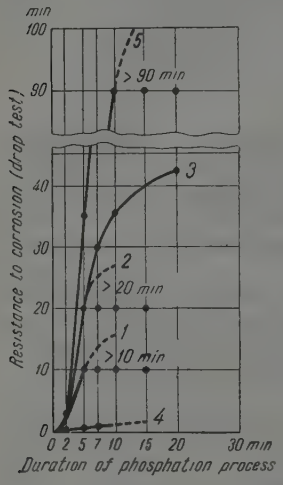

Fig. 1. Protective properties of phosphate films as dependent on the duration of the phosphating process (at 98°C) and subsequent treatment (U-3 steel). *1*, phosphating (Ph); *2*, Ph+No.1 treatment with chromate (Ch.); *3*, Ph+impregnation with vaseline solution (V); *4*, Ph+No. 3 chromate treatment; *5*, Ph+Ch. No. 1+V

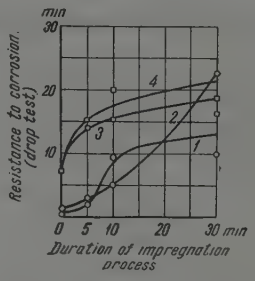

Fig. 2. Corrosion resistance of U-3 polished steel specimens as affected by the length of impregnation of the phosphate layer with a 9 per cent solution of $K_2Cr_2O_7$ (at 90°C). *1*, *2*, phosphatization during 5 minutes; *3*, *4*, phosphatization during 10 minutes

tent, appearance of «spots», excessive grain size, *etc.*).

Our experiments, which were made on specimens of different carbon steels and then on various finished steel parts, were carried out at $t = 96$ and 98 +0.2° in a four-litre electrolytic bath with an automatic temperature control and containing a normal phosphating solution, 30—32 g/l Mn—Fe phosphate, the ratio of the general acidity (K_g) to the free acidity (K_{free}) being maintained at $\eta = 7—7.5$.

The resistance of the phosphate films to corrosion was determined by our express method—a test drop with a cathode (copper) indicator (reagent No.2), the drop undergoing a sharp change in colour when the phosphate film is destroyed (start of corrosion).

Numerous experiments fully confirmed our anticipations as to the results of accelerated (incomplete) phosphating treatment (Fig. 1). The resistance of the film to corrosion increases especially rapidly in the first stage of the phosphating process, attaining a value nearly equal to that for specimens of normal—complete—phosphating.

Further, by means of a large number of experiments on increasing the protective properties of accelerated phosphate films after their impregnation with

* Solutions of lanoline or vaseline are likewise employed to increase the resistance to corrosion of ordinary phosphate films.

solutions of $K_2Cr_2O_7$ (90 g/l) of varying composition (altered by adding different amounts of CrO_3 (5—20 g/l)), carried out at temperatures of 20, 50, 90 and 95°C and for periods of 5, 10 and 30 min., it was established that chromate treatment markedly increases the resistance of these films to corrosion (curve *2*, Fig. 1). The optimum conditions of this treatment (impregnation) are as follows: concentration, 8—10 per cent $K_2Cr_2O_7$ and $t = 80$—90°C. More acid (from 1 per cent CrO_3 or 0.2 per cent H_3PO_4) and more basic (from 0.001 to 0.1 *M* NaOH) solutions of chromate gave less favourable results. Curve *4* in Fig. 1 shows that a 9 per cent $K_2Cr_2O_7 + 1$ per cent CrO_3 solution completely destroys the protective film.

A big rôle in chromate treatment is also played by the time of impregnation (Fig. 2), the optimum conditions here being 10—30 min. (on the average 20 min.).

Finally, additional impregnation with organic substances (solutions of lanoline or vaseline in gasoline or turpentine) further increases the protective properties of both the phosphate layer (curve *3*, Fig. 1) and, still more markedly, the phosphate-chromate layer (curve *5*, Fig. 1), bringing them up to the required corrosion resistance standard (different durations of phosphate treatment were tried).

On the basis of the results obtained we developed a new method, consisting in the utilization of more uniform, thin and porous films created by a sharply curtailed (before evolution of hydrogen stops) but very definite (5—10 min.) phosphating regime, and the subsequent impregnation of the film with inorganic inhibitors (chromates) and solutions of organic substances (lubricants).

A detailed technological scheme of the new method of express phosphating was likewise worked out. After testing on a large consignment of manufactured parts (30 000 details) the new method was incorporated in the serial production of a large factory. The following points give its substantial characteristics.

a) The process is confined within a strictly definite, previously fixed, interval, making it possible to work on a strict time schedule with the available equipment, and lends itself to mechanization and automatic control.

b) A three to fivefold increase in the speed of the process.

c) A three to fourfold curtailment in consumption of the phosphating reagents (MAGEF).

d) High quality of the smooth, finely crystalline (dull) coating, which come up in corrosion resistance to the peacetime standards.

d) A three to fivefold curtailment in overall spoilage.

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Received
20. IV. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

CHIMIE PHYSIQUE

TRAVAIL ET CHALEUR D'ADSORPTION

Par A. V. KISSELEV

(Présenté par M. M. Dubinin, de l'Académie, le 2.I.1945)

Le problème du travail et de la chaleur d'adsorption n'a pas été jusqu'à présent systématiquement analysé en général. Il n'existe que des formules isolées pour les cas particuliers examinés indépendamment l'un de l'autre. L'analyse thermodynamique détaillée faite par Williams [1] n'a pour objet que la chaleur d'adsorption de la substance pure.

D'après certains auteurs l'interprétation thermodynamique ne serait pas applicable à l'adsorption sur les substances solides [2]. Cependant Gibbs [3] a trouvé pour les phases homogènes et pour les surfaces les équations fondamentales à l'aide desquelles il a réussi à déterminer les changements de l'énergie libre et totale de chaque processus d'adsorption [4]. Après avoir formulé à l'aide des équations fondamentales les énergies libres à l'état initial et à l'état final (avant et après la sorption), il n'est pas difficile de représenter leur différence par une formule qui se prêterait facilement au calcul. Si l'adsorbant 1 reste invariable, c'est-à-dire, si son potentiel chimique μ_1 est constant, le travail d'adsorption du gaz pur ou de la vapeur 2

$$A_a = s\,(\sigma_1 - \sigma + \Gamma_2 RT \ln \tfrac{p_0}{p}) \qquad (1)$$

s étant la surface, σ_1 et σ les tensions superficielles avant et après l'adsorption, p_0 et p les fugacités (approximativement les pressions de vapeur partielles de la substance adsorbée 2 à l'état initial et à l'état d'équilibre avec l'adsorbant, et Γ_2 — adsorption par une unité de surface *.

La formule (1) démontre en toute évidence que le travail d'adsorption et le changement de tension superficielle présentent en général de grandes différences: le travail d'adsorption dépend considérablement de l'état initial que nous avons choisi; ce que l'on ne prend pas ordinairement en considération.

En intégrant l'équation de Gibbs ([1],[4],[5]), on peut exclure de (1) $\sigma_1 - \sigma$ et obtenir une formule équivalente pour A_a, qui serait applicable pour les cas d'adsorption sur les adsorbants solides

$$A_a = \int_0^{x_2} RT \ln \frac{\sigma}{p}\, dx_2 \qquad (2)$$

d'où les formules suivantes pour la chaleur intégrale

$$Q_a = S\left\{\sigma_1 - \sigma - T\left[\frac{\partial(\sigma_1 - \sigma)}{\partial T}\right]_p\right\} = \int_0^{x_2} RT^2 \left(\frac{\partial \ln p}{\partial T}\right)_{x_2} dx^2 \qquad (3a,b)$$

$$Q_a = S\left\{\sigma_1 - \sigma - T\left[\frac{\partial(\sigma_1 - \sigma)}{\partial T}\right]_{\Gamma_2} + \Gamma_2 RT^2 \left(\frac{\partial \ln p}{\partial T}\right)_{\Gamma_2}\right\} \qquad (4)$$

Les équations (3,a,b) étaient obtenues par Williams [1], Cassel [5] a donné la formule (4) (pour le cas particulier de l'isotherme linéaire).

* Comme l'adsorption Γ_1 disparaît de l'équation de Gibbs lorsque $d\mu_1 = 0$, $-d\sigma = \Sigma_i \Gamma_i d\mu_i$ la surface géométrique déterminant Γ_2 peut être choisie arbitrairement. Il est nécessaire de prendre une surface physique de séparation; dans ce cas $x_2 = s\Gamma_2$ serait l'excès du nombre de molécules-grammes de la substance 2 dans un volume adjacent à la surface s et contenant tout le domaine de l'hétérogénéité sur le nombre de celles-ci à l'intérieur de la phase homogène.

Lorsque la saturation par les vapeurs est complète

$$A_a^s = s(\sigma_1 - \sigma_s) = \int_0^{x_2^s} RT \ln \frac{p_s}{p}\, dx_2 \qquad (5a, b)$$

σ_s étant la tension superficielle à la surface de séparation adsorbant — vapeur saturée, et x_2^s — adsorption des substances se trouvant dans la vapeur saturée, dont la pression de vapeur est p_s, et

$$Q_a^s = s\left[\sigma_1 - \sigma_s - T\frac{d(\sigma_1 - \sigma_s)}{dT}\right] = \int_0^{x_2^s} RT^2 \left(\frac{\partial \ln p}{\partial T}\right)_{x_2} dx_2 \qquad (6a, b)$$

Dans le cas, où l'adsorbant est mouillé par le liquide pur 2,

$$A_a^l = s(\sigma_1 - \sigma_{1-2}) = A_a^s + s\sigma_2 \qquad (7a,b)$$

où σ_{1-2} — tension entre les phases adsorbant — liquide, et σ_2 — tension superficielle du liquide, et

$$Q_a^l = s\left[\sigma_1 - \sigma_{1-2} - T\frac{d(\sigma_1 - \sigma_{1-2})}{dT}\right] = Q_a^s - x_2^s L_2 + s\left(\sigma_2 - T\frac{\partial \sigma_2}{\partial T}\right) \qquad (8a, b)$$

L_2 étant la chaleur latente de vaporisation du liquide 2. La valeur $Q_a^l - (Q_a^s - x_2^s L_2)$ représente la chaleur de mouillage de l'adsorbant saturé par les vapeurs. Celle-ci mesurée, on peut définir s pour les adsorbants non-poreux (7). En ce qui concerne les adsorbants poreux on peut négliger les termes contenant σ_2 (en cas de mouillage complet).

Pour le travail différentiel d'adsorption du gaz nous avons

$$A_{x_2} = RT \ln \frac{p_0}{p} \qquad (9)$$

ou, lorsque l'état initial de la substance 2 est une phase condensée

$$A_{x_2} = RT \ln \frac{p_s}{p} \qquad (10)$$

La dernière formule fut adoptée par Goldmann et Polanyi (8).

La chaleur différentielle d'adsorption du gaz ou de la vapeur peut être représentée par les équations suivantes:

$$Q_{x_2} = RT^2 \left(\frac{\partial \ln p}{\partial T}\right)_{x_2} = -RT^2 \left(\frac{\partial \ln x_2}{\partial T}\right)_p \Big/ \left(\frac{\partial \ln x_2}{\partial \ln p}\right)_T \qquad (11a, b)$$

D'où il suit que la même chaleur d'adsorption mesurée par un procédé calorimétrique peut être formulée à l'aide de coefficient de température «isostérique» (11a), des valeurs ayant rapport à l'équilibre d'adsorption, ainsi que par le coefficient «isobarique» («isopicnique») (11b) de ces valeurs. Il n'y a aucune raison de désigner (11a) et (11b) comme la chaleur d'adsorption «isostérique» (9) et «isobarique» ou «isopicnique» (10).

$x_2 s$ étant égale à $\Gamma_2 = pG_2/RT$, où $G_2 = -(\partial\sigma/\partial p)_T$ représente l'activité superficielle, on peut donner à (11a) et (11b) encore deux formes équivalentes qui se prêteraient au calcul sur les surfaces liquides:

$$Q_{\Gamma_2} = RT\left[1 - \frac{T}{G_2}\left(\frac{\partial G_2}{\partial T}\right)_{\Gamma_2}\right] = RT\frac{1 - \frac{T}{G_2}\left(\frac{\partial G_2}{\partial T}\right)_p}{1 + \frac{p}{G_2}\left(\frac{\partial G_2}{\partial p}\right)_T} \qquad (12a, b)$$

dont la première contient un coefficient isostérique et la deuxième — isobarique (isopicnique) *.

Le cas, où l'adsorption d'une substance dissoute a lieu, est plus compliqué. Si un mélange des gaz est absorbé, les valeurs d'adsorption de chaque com-

* Pour le cas particulier d'une zone linéaire de l'isotherme nous avons les équations:

$$Q_{\Gamma_2} = -RT^2\left(\frac{\partial \ln \Gamma_2}{\partial T}\right)_p = RT\left[1 - \frac{T}{G_2}\left(\frac{\partial G_2}{\partial T}\right)_p\right] =$$

$$= \frac{1}{\Gamma_2}\left\{\sigma_1 - \sigma - T\left[\frac{\partial(\sigma_1 - \sigma)}{\partial T}\right]_p\right\} \qquad (11b', 12b', 3a')$$

Les formules (11b') et (12b') sont données par Rehbinder (10), et la formule (3a')—par Cassel (6).

posant peuvent augmenter simultanément, au moins dans le domaine des pressions faibles; les formules (1)—(11) restent les mêmes, on ajoute seulement les membres ayant rapport aux autres composants. En ce qui concerne le cas, où l'adsorption d'une substance dissoute d'une solution condensée a lieu, les valeurs d'adsorption des composants dépendent des conditions du déplacement. Par suite, pour une solution à deux composants

$$v_3 x_3 + v_2 x_2 = 0;\ x_2 = -\beta x_3 \qquad (13a, b)$$

où v_3 et v_2 sont les volumes en molécules, et β — leur rapport. Comme les substances 2 et 3 ne pénètrent pas à l'intérieur de l'adsorbant, il est utile de maintenir la même position de la surface géométrique et la même définition pour l'adsorption ([11]).

En ce qui concerne le changement de l'énergie libre pendant le mouillage de l'adsorbant pur par la solution, nous trouvons selon les équations fondamentales de Gibbs

$$\Delta Z = s(\Delta\sigma + \Gamma_2 \Delta\mu_2 + \Gamma_3 \Delta\mu_3) + n_2' \Delta\mu_2 + n_3' \Delta\mu_3 \qquad (14)$$

où n_2' et n_3' sont les nombres de molécules-grammes qui restent dans un volume de la solution homogène (jusqu'à la surface s). x_2 exprimé par x_3 d'après (13b) et μ_2 par μ_3 d'après l'équation Gibbs—Duhem, nous avons dans le cas d'une solution idéale:

$$A = -\Delta Z + s\left[\sigma_1 - \sigma + \Gamma_3 RT\left(\ln\frac{N_3^\circ}{N_3} - \beta \ln\frac{1-N_3^\circ}{1-N_3}\right)\right] + \\ + n_3' RT \ln\frac{N_3^\circ}{N_3} + n_2' RT \ln\frac{1-N_3^\circ}{1-N_3} \qquad (15)$$

où N_3 — portion mol. de la substance 3, l'adsorption de laquelle est mesurée. Le premier membre exprime le changement de l'énergie libre dans le domaine de l'hétérogénéité, le deuxième — celui dans une solution homogène (travail de dilution A'). Si la solution n'est pas idéale, il faut introduire le coefficient d'activité.

Dans le cas, où l'adsorption sur les adsorbants solides a lieu, il faut exclure $\Delta\sigma$ en intégrant l'équation de Gibbs depuis $\Gamma_3 = 0$, $\sigma = \sigma_{1-2}$ (adsorbant — solvant pur 2) jusqu'à Γ_3, σ (adsorbant — solution). Etant donné que $\sigma_1 - \sigma = \sigma_1 - \sigma_{1-2} + \sigma_{1-2} - \sigma$, et que d'après (7a) s $(\sigma_1 - \sigma_{1-2})$ représente le travail A_2 de mouillage par le solvant pur 2,

$$A = A_2^s + \int_0^{x_3} RT\left(\ln\frac{N_3^\circ}{N_3} - \beta \ln\frac{1-N_3^\circ}{1-N_3}\right) dx_3 + A'. \qquad (16)$$

D'où on trouve pour la chaleur d'adsorption d'une substance dissoute (en négligeant $d\beta/dT$)

$$Q \cong Q_2^s + \int_0^{x_3} \left(1 + \beta\frac{N_3}{N_2}\right) RT^2 \left(\frac{\partial \ln N_3}{\partial T}\right)_{x_3} dx_3 + Q' \qquad (17)$$

où Q_2^s — chaleur de mouillage par un solvant, et Q' — chaleur de dilution.

De cette façon le travail (ou la chaleur) d'adsorption d'une substance dissoute est égal à la somme: travail (chaleur) de mouillage de l'absorbant par le solvant, travail (chaleur) d'adsorption de la substance dissoute et du déplacement du solvant par celle-ci, et travail (chaleur) de dilution. Pour les solutions idéales $Q' = 0$.

Si aucun changement de concentration n'a lieu pendant l'adsorption (par exemple, en présence de la substance pur 3), (15) est transformée en

$$A = s(\sigma_1 - \sigma)_{N_3} \qquad (18)$$

et la chaleur d'adsorption peut être exprimée d'une façon analogue à (3a)

$$Q = s\left\{\sigma_1 - \sigma - T\left[\frac{\partial(\sigma_1 - \sigma)}{\partial T}\right]_{N_3}\right\} \qquad (19)$$

Pour le travail et la chaleur différentiels d'adsorption de la substance dissoute nous trouvons respectivement

$$A_{x_3} = RT\left(\ln\frac{N_3^s}{N_3} - \beta\ln\frac{1-N_3^s}{1-N_3}\right) \qquad (20)$$

et

$$Q_{x_3} \cong \left(1+\beta\frac{N_3}{N_2}\right)RT^2\left(\frac{\partial \ln N_3}{\partial T}\right)_{x_3} =$$

$$= -\left(1-\beta\frac{N_3}{N_2}\right)RT^2\left(\frac{\partial \ln x_3}{\partial T}\right)_{N_3} \Big/ \left(\frac{\partial \ln x_3}{\partial \ln N_3}\right)_T \qquad (21a, b)$$

Pour une zone linéaire de l'isotherme

$$Q_{x_3} \cong -\left(1+\beta\frac{N_3}{N_2}\right)RT^2\left(\frac{\partial \ln x_3}{\partial T}\right)_{N_3} \qquad (22)$$

Les formules ultimes pour les concentrations très faibles lorsque $\beta(N_3/N_2) \ll 1$:

$$A_{x_3\to 0} = RT\ln\frac{N_3^s}{N_3} \qquad (23)$$

$$Q_{x_3\to 0} = RT^2\left(\frac{\partial \ln N_3}{\partial T}\right)_{x_3} \cong -RT^2\left(\frac{\partial \ln x_3}{\partial T}\right)_{N_3} \qquad (24a, b)$$

On pourrait encore trouver les formules exprimant la chaleur d'adsorption de la substance dissoute en partant de la valeur de l'activité superficielle $G_3 = -(\partial\sigma/\partial N_3)_T$. Pour cela il faut remplacer $\Gamma_3 = x_3/s$ par $\Gamma_3^{(2)}$ de la surface géométrique correspondante pour laquelle $\Gamma_2 = 0$, car c'est seulement $\Gamma_3^{(2)}$ qui est égal à $N_3 G_3 RT$. Guggenheim et Adam ([11]) ont souligné les inconvénients d'un choix pareil de la position de la surface géométrique*.

Le choix de l'état initial de la solution a une grande importance pour la détermination du travail d'adsorption des substances dissoutes, comme cela a lieu dans les cas des substances pures.

L'expérience montre que x_3 diminue ordinairement lorsque T augmente (ou $(\partial \ln N_3/\partial T)_{x_3} > 0$). Ce fait aboutit à la formation des chaleurs de mouillage positives. Ceci est confirmé par l'expérience**.

Etant donné que la définition expérimentale des chaleurs d'adsorption ne présente aucune difficulté particulière, on peut vérifier les postulats donnés à l'aide des formules énoncées. Par conséquent, les mesures des chaleurs d'adsorption des substances dissoutes prennent un intérêt particulier, si ces mesures sont faites parallèlement aux isothermes de l'adsorption elle-même à différentes températures; il est de même dans les cas où le mouillage par les solutions concentrées a lieu, et où l'adsorption x_3 passe par un maximum.

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Manuscrit reçu
le 23. XII. 1945.

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* Rehbinder ([10]) a eu recours, pour l'analyse de l'adsorption des substances dissoutes en présence d'une courbure considérable des isothermes, à la formule (12b′) qui n'est applicable qu'à la zone linéaire de l'isotherme. Les chaleurs d'adsorption qu'il avait calculées accroissent rapidement (de dix fois) à mesure que la température augmente. Ordinairement la chaleur d'adsorption ne dépend que peu de la température.

** Heyne et Polanyi ([12]) ont exprimé l'affinité d'adsorption d'une substance dissoute par la formule $a_x = RT\ln(c_g/c)$, où c_g est la concentration d'une solution saturée et c—celle d'une solution en équilibre. Cet état initial rend impossible une solubilité illimitée, et établit le fait qu'une dépendance prononcée a lieu entre a_x et les qualités de c_g. Dans les cas examinés par les auteurs, a_x augmente en même temps que T, grâce à l'accroissement rapide de c_g avec T. Par conséquent, l'adsorption ultime (lorsque $a_x = 0$) augmente aussi avec T, tandis que l'adsorption diminue lorsque T s'élève dans l'intervalle de l'expérience. L'accroissement de a_x avec T a amené les auteurs à la constatation des chaleurs négatives, ce qui est en désaccord avec les données de l'expérience. En outre, dans la formule qui représente a_x, le déplacement du solvant n'est pas pris en considération.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

CHIMIE PHYSIQUE

L'ÉLECTROPHORÈSE DU LYSOCIME PAR LA MÉTHODE DE TISELIUS

Par A. PASSYNSKI et T. KASTORSKAYA

(Présenté par A. N. Bach, de l'Académie, le 8.XII.1944)

L'analyse électrophorétique par la méthode de Tiselius ([1],[2]) des albumens séparées ou des mélanges d'albumens a acquis en ces dernières années une importance considérable. Cette méthode permet de contrôler l'homogénéité de la solution d'albumen, de déterminer le nombre de composants dans un mélange d'albumens et de mesurer la mobilité électrophorétique de chacun de ces composants comme fonction de constitution du milieu, et ces études peuvent être faites

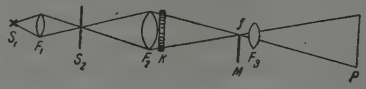

Fig. 1

directement sur le mélange d'albumens sans séparation fractionnée préalable. La méthode des mesures électrophorétiques précises des mélanges d'albumens mise au point par Tiselius est basée sur l'emploi de potentiels élevés (7—10 V/cm), ce qui est 6—10 fois supérieur au voltage utilisé jusqu'alors et a permis d'augmenter la mobilité des différents composants pour permettre leur observation à l'aide d'un appareillage optique spécial quelconque, basé généralement sur le principe de Viner; entre autres, Tiselius a indiqué les moyens d'éviter les nombreuses causes d'erreur qui ont lieu au cours de l'électrophorèse des albumens quand on emploie des potentiels élevés.

Dans le présent travail nous avons construit un appareil pour l'électrophorèse des albumens par la méthode de Tiselius, appareil muni d'un appareillage optique de Tepler («Schlierenmethode»), et après une série de mesures de contrôle effectuées sur des solutions d'albumens d'oeuf, mesures qui nous ont montré une correspondance complète de nos résultats avec ceux de Tiselius, nous avons appliqué la même méthode à l'étude du lysocime dont les valeurs électrophorétiques étaient inconnues jusqu'à ce jour.

La figure 1 représente le schéma optique de l'appareillage. Le rayon de lumière issu de la lampe S_1 de 200 watt traverse l'orifice de la cage de protection munie d'une double paroi, refroidie par un courant d'eau, et est rassemblée par le condensateur F_1 sur la fente étroite S_2. L'objectif F_2 se trouve de la fente S_2 à une distance double de la distance focale (distance optique $S_2F_2 = 60$ cm) et rassemble la lumière au point f. Le récipient électrophorétique K se trouve placé dans le

thermostat (muni de doubles parois avec des petites fenêtres parallèles) le plus près possible de l'objectif F_2. L'image du récipient K est projetée sur l'écran P (plaque photographique) au moyen de l'objectif F_3 (F=30 cm). Au point focal f est disposé le rebord de la lame tranchante M, qui peut être déplacée grâce à une vis micrométrique. En cas de liquide homogène dans le récipient K l'image sur l'écran est régulièrement éclairée, mais si une surface de contact entre deux liquides apparaît dans le récipient K sur la route du rayon lumineux, le rayon est dévié par cette surface et est intercepté par la lame M, et par suite, une bande sombre apparaît sur l'écran dans un emplacement correspondant. La distance entre le point focal f et l'objectif F_3 comporte quelques centimètres, ce qui fait que l'augmentation comporte 1.05—1.10. L'exposition eu lieu pendant 40—50 secondes.

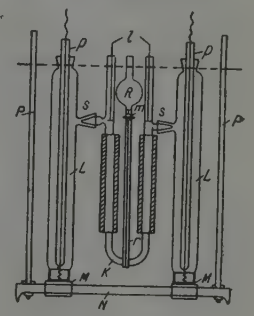

Fig. 2

Le récipient électrophorétique K est constitué par un tube en U (figure 2) aux parois paralèlles fixées par un ciment spécial. Le récipient est rempli de la solution étudiée au moyen du capillaire étroit r, qui vient s'aboucher par en bas au tube en U et amène le liquide du réservoir R. Les joints étanches s réunissent le récipient K aux deux récipients à électrodes L d'un volume de 500 cm³. Les récipients contenant les électrodes sont fixés par le ciment de Mendéléev dans les godes en étain M, mobiles le long du support N. Le support N est muni de deux tiges en étain P, au moyen desquelles l'appareil se laisse plonger en entier dans le thermostat ou s'en laisse retirer. Les électrodes sont constitués par un fil d'argent épais en forme de spirale, recouvert de AgCl, et en les plongeant au fond des récipients on les faisait passer par les tubes en verre p. Les récipients à électrode sont remplis dans leur partie inférieure d'une solution concentrée de KCl; le volume restant des récipients à électrode et tout le récipient électrophorétique K sont remplis d'une solution-tampon de la substance par rapport à laquelle on procède à l'électrophorèse de la solution d'albumen. La solution d'albumen étudiée (10 à 12 cm³ environ) est versée, le robinet m étant fermé, dans le récipient R, et l'appareil, étant ainsi rempli, est plongé en entier dans le thermostat. Après une heure, une heure et demi le robinet m est ouvert, et la solution étudiée est versée avec de grandes précautions sous la solution-tampon. L'opération est contrôlée optiquement. Quand la surface de contact atteint la partie moyenne du tube en U, le robinet m est fermé, et on applique aux électrodes une tension de 400—800 V provenant d'un redresseur à kenotron stabilisé. Un voltmètre électrostatique précis permet de mesurer la chute de potentiel sur les électrodes l dans le tube en U. On photographie toutes les 20—30 minutes les positions successives des surfaces de contact en mouvement, suivant les conditions de l'expérience.

Le lysocime est préparé à partir de l'albumine d'oeuf frais par la méthode de Bordet modifiée, et est purifié par une solution d'alcool à 80 pour cent (d'après la méthode de Bouyanovskaya [3]). Ensuite la solution de lysocime purifié d'une concentration de 0.8 pour cent est dialysée soigneusement dans un réfrigérateur à travers une membrane collodion par rapport à l'eau distillée. On ajoute ensuite à la solution de lysocime une solution-tampon de phosphate de concentration 0.02 M, et on procède à la détermination de pH (en utilisant une électrode en verre) et de la conductibilité électrique. On équilibre les pH de la solution de lysocime et de la solution voisine avec une précision de 0.01—0.02 pH. La conductibilité électrique de la solution voisine est de 2—3 pour cent plus élevée que celle de la solution de lysocime. Après la fin de l'expérience d'électrophorèse on contrôle la constance du pH de la solution de lysocime; la variation du pH au cours de l'expérience, c'est-à-dire pendant $1^1/_2$—2 heures, ne dépasse pas 0.03 pH. Les mesures étaient faites à la température de 20° C±0.01.

Les résultats sont indiqués dans le tableau (malheureusement, des circonstances

extérieures on limité le nombre de nos expériences avec les solutions de lysocime). Le tableau considéré indique que dans la région neutre correspondant à un pH de 6—8 le lysocime se comporte comme un corps homogène au point de vue électrochimique (du moins pour une durée d'expérience d'une heure et demi et un coefficient de potentiel de 7 V/cm), et ce n'est que dans la zone acide, pour un pH = 5, que l'on peut observer l'apparition d'un second composant.

pH		Conductibilité électrique		Coefficient potentiel V/cm	Mobilité électrophorétique par $\frac{cm^2 \cdot 10^5}{V \cdot sec}$
solution-lysocime	solution-tampon	solution-lysocime	solution-tampon		
4.94	4.92	2.07	2.10	10.0	1)+2.5 2)+0.56
6.10	6.10	3.12	3.21	6.5	—0.48
7.81	7.83	5.96	6.10	7.0	—0.60

Le point isoélectrique du composant principal du lysocime correspond au pH = 6.0, et l'inclinaison de la courbe de mobilité dans la région du point isoélectrique est $\frac{du}{dpH_0} = 2.5 \cdot 10^{-5}$. Cette position du point isoélectrique pour pH = 6.0 correspond à la propriété basique du lysocime, ce que confirment les résultats de l'analyse directe des acides aminés entrant dans la constitution du lysocime (4). Pour le pH des solutions purifiées et dialysées du lysocime, ce qui correspond au point isoionique de la solution, nous avons trouvé dans nos expériences la valeur de 5.92—6.0 (mesurée avec une électrode en verre); cette valeur correspond bien au résultat indiqué plus haut. Dans ses études qualitatives sur l'activité du lysocime pour différentes valeurs du pH, Chabordine (5) a montré que le maximum de l'activité lytique du lysocime correspond à un pH de 6.4. Cette grandeur se rapproche également du point isoélectrique du lysocime que nous avons déterminé, si l'on tient compte des inexactitudes inhérentes à des mesures semblables. Enfin, d'après les données de Chabordine, l'adsorption du lysocime par des bactéries sensibles est considérablement plus élevée pour un pH de 5.9, que pour un pH de 7.1. La comparaison des données précitées permet la conclusion logique que le maximum de l'activité lytique du lysocime, et également adsorption maximum du lysocime par des bactéries sensibles, correspond probablement au point isoélectrique du lysocime, ou du moins est étroitement lié à sa valeur; cette circonstance est importante pour comprendre le mécanisme d'action du lysocime.

* * *

Dans le travail de A. Passynski at V. Plaskéev (6) il a été établi que le poids moléculaire du lysocime est de $M = 13\,000$ et le coefficient de diffusion $D_{20,w} = 8.6 \cdot 10^{-7}$. Pendant la publication de la présente étude et du travail précité, G. Alderton, W. Ward et H. Fewold (7) ont fait paraître un article relatif à leurs études du lysocime, obtenu par une autre méthode—par adsorption sur le bentonite avec lavement consécutif; ils ont obtenu pour le produit considéré les valeurs de $M = 14\,000$—$17\,000$ (par la méthode d'ultra-centrifugation) et $D_{20,w} = 11.2 \cdot 10^{-7}$, ce qui correspond à nos résultats, si l'on tient compte de la différence des produits étudiés. Les auteurs américains ont trouvé que le point isoélectrique du lysocime était de $IP \simeq 10.5$, mais dans leur travail les points isoélectriques qu'ils ont déterminés, en se servant d'acétates et de phosphates comme tampons, s'en écartent notablement et ont une valeur de $IP \simeq 7.0$, ce qui correspond bien également aux valeurs que nous avons obtenues avec

des phosphates comme substances-tampons (IP=6.0). La différence entre les valeurs de IP=6—7 et 10.5 pour les points isoélectriques s'explique vraisemblablement par des différences dans la constitution des substances-tampons (véronal, étanolamine et autres) que les auteurs américains ont utilisées pour leurs expériences, et par la formation de complexes entre le lysocime et les substances-tampons en solution. Au point de vue moléculaire et électrochimique les divers échantillons du lysocime se sont montrés homogènes, ce qui correspond à nos résultats. Les hypothèses d'une structure complexe du lysocime ([8]) se trouvent ainsi donc vraisemblablement confirmées par les expériences précises de G. Alderton, J. Lewis et H. Fewold ([9]).

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Manuscrit reçu
le 8. XII. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

TECHNOLOGIE CHIMIQUE

INFLUENCE DES COMPOSANTS SUR LA TEMPÉRATURE DE MOLLIFICATION DES MATIÈRES BITUMINEUSES

Par A. V. SOLOVIEV

(Présenté par V. G. Chlopin, de l'Académie, le 12. III. 1945)

L'influence des composants en poudre et des composants filamineux sur la température de mollification, la dilatabilité et la solidité des matières bitumineuses sont connues. Les propriétés physico-chimiques des matières bitumineuses peuvent être réglées à l'aide des composants dans le but de construire des revêtements anticorrosifs et hydro-isolants avec un maximum de stabilité et d'obtenir des masses plastiques et des bétons les plus durables.

Il serait d'un grand intérêt d'élucider quelles sont les causes provoquant la variation des qualités physico-chimiques des matières bitumineuses en fonction de la quantité de leurs composants, et quelle est la régularité de cette relation.

L'analyse théorique du phénomène envisagé nous conduit à admettre que celui-ci dépend des forces moléculaires adhésives entre les bitumes et les composants. L'adhésion des bitumes avec la plupart des corps solides est plus forte que la cohésion entre les particules du bitume. De cette façon en ajoutant au bitume des composants moulus fins nous parvenons à augmenter l'association interne, la température de mollification et la solidité des matières bitumineuses.

L'accroissement de la température de mollification et de la solidité mécanique des matières bitumineuses présent une fonction de la surface active où ont lieu le contact et l'adhésion du bitume avec ses composants. La surface active du composant est d'autant plus grande que la mouture de ceux-ci est plus fine. Par conséquent, l'augmentation de la température de mollification et de la solidité de la matière bitumineuse, qui a lieu lorsqu'on introduit des composants dans la matière bitumineuse, est d'autant plus fine que la mouture du composant est plus fine. Si la finesse de mouture du composant reste la même, on peut constater la régularité suivante.

L'accroissement de la température de mollification de la matière bitumineuse est en proportion directe au carré de la quantité du composant introduit et en proportion inverse à la quantité du bitume contenu. Cette régularité est exprimée par l'équation suivante:

$$T - T_0 = k\frac{m^2}{n}$$

T étant la température de mollification du bitume; T_0—celle de la matière bitumineuse contenant le composant; m—quantité de composant en poudre contenu dans la matière bitumineuse (pourcentage en poids); n—quantité de bitume contenu dans la matière bitumineuse (pourcentage en poids); k—coefficient de proportionalité.

Le coefficient de proportionalité de chaquetype de bitume et de chaque espèce de composant est d'une valeur définie, et il dépend de la température de mollification du bitume, de la composition chimique du bitume et du composant, de la finesse de mouture et de la propriété adsorptive du composant. Le coefficient de proportionalité est déterminé expérimentalement.

A présent comparons les données expérimentales se rapportant à la température de mollification des matières bitumineuses, préparées à partir de la portion organique du goudron acide et de la poudre de calcaire, avec les données théoriques obtenues à l'aide de l'équation ci-dessus mentionnée.

On obtenait la masse organique bitumino-résineuse par séparation des acides du goudron acide (usine Tchkalov) au moyen de lavage quadruple par l'eau en faisant bouillir le produit. La température de mollification de cette masse est égale à 32°C. En introduisant dans cette masse du calcaire moulu, passé au travers du tamis à mailles de 0.25 mm de diamètre, ou bien de la chaux éteinte, passée au travers du même tamis, on obtient des matières bitume-émail dont la température de mollification est plus élevée que celle de la portion organique du goudron acide.

Le tableau représente les données expérimentales et celles obtenues par calcul se rapportant à la température de mollification des matières bitume-calcaire et bitume-chaux. Le coefficient k pour la portion organique du goudron acide et pour le composant en calcaire est égal à 0.28. En ce qui concerne la portion organique du goudron acide et du composant en chaux (chaux éteinte) il est égal à 2.66.

Influence sur la température de mollification de la portion organique du goudron acide, des additions de calcaire et de chaux éteinte avec des particules ayant moins 0.25 mm de diamètre

Teneur en pour cent en poids			Température de mollification en °C d'après la méthode «anneau et sphère»	Coefficient	Température de mollification calculée d'après la formule (1) en °C
Portion organique obtenue du goudron acide quatre fois lavé	Calcaire avec des particules ayant moins 1.25 mm de diamètre	Chaux éteinte avec des particules ayant moins 0.25 mm de diamètre			
100	—	—	32	0.28	32
70	30	—	33	0.28	35.6
60	40	—	39.5	0.28	39.46
50	50	—	45	0.28	46
40	60	—	52	0.28	57.2
30	70	—	80	0.28	77.7
90	—	10	41	2.66	39.95
80	—	20	46	2.66	45.3
70	—	30	63	2.66	66.2
65	—	35	86	2.66	82.13

Les données expérimentales et théoriques se rapportant aux températures de mollification des matières bitume-calcaire et bitume-chaux sont voisines. Il y a certaines divergences entre ces deux valeurs, qui dépendent de la propriété adsorptive sélective des composants par rapport aux composants principaux du bitume: résines, asphaltes, huiles. Ces divergences sont tout à fait insignifiantes. Par conséquent, l'équation énoncée peut être largement utilisée lorsqu'il s'agit de la définition à l'aide de la température de mollification des composants des mastiques de bitume. La quantité du composant capable d'être cimentée par le bitume est limitée. La quantité ultime de la substance minérale qui peut être ajoutée au bitume dépend de la densité de cette substance. Plus la substance du composant est dense, moins le besoin en matière cimentante est grand; ce besoin augmente avec la distance entre les particules du composant.

Il est fort probable que l'équation énoncée pourrait être appliquée à d'autres matières plastiques.

Manuscrit reçu
le 12. III. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

GEOLOGY

ON THE PRINCIPAL STRUCTURAL ELEMENTS AND POSSIBLE FUTURE OIL AND GAS RESOURCES OF THE SOUTHERN BORDER OF THE EUROPEAN PART OF THE USSR

Par I. O. BROD

(Communicated by P. I. Stepanov, Member of the Academy, 8. VI. 1945)

The problems involved in the creation of the «Great Donetz Basin» and in the search for new large oil-bearing zones within the vast plains in the south of the USSR are intimately related to the question as to the age and structure of the basement of the southern part of the Russian platform ([1],[2]). Numerous gas occurrences known in the near-Caspian, Kalmyk-Salsk, North Caucasian and Crimean steppes, as well as in the Melitopol region, are usually considered as gas accumulations lying at shallow depths and having a purely local significance. As a matter of fact, the basement of the above-listed regions being made up of folded structures of different age, the prospects of economic gas and oil possibilities of these regions are to be viewed in a quite different light.

In the central regions of the Russian platform drilling has revealed the occurrence of the pre-Cambrian basement at depths not exceeding 2000 m. In the southeast the area of an uplifted occurrence of pre-Cambrian rocks stretches in a belt from the Voronezh massif to the Ufa plateau ([3]). The southern slope of the uplifted zone, well traceable from data of magnetic and gravimetric surveys, plunges non-uniformly towards the foot of the Caucasian and Crimean mountains. This plunge is interrupted by great uplifts, as a result of which the pre-Cambrian has at places been brought out to the day's surface (Azov-Podolia massif).

There is no doubt as to the subdivision of the basement east of the above-named region either, where the presence of a buried range postulated as early as by A. P. Karpinsky ([4]), was confirmed by data of subsequent geophysical research ([5]). F. F. Golynetz and N. G. Uspenskaya, who explored this region, considered the buried structure, in accordance with the views of A. P. Karpinsky, as a connecting link between the Mangyshlak mountains and the folds of the Donetz Basin ([2],[6]). The geophysical exploration of the subsequent decade, generalized in 1940 by E. E. Fatiadi and after him by A. A. Borissov, showed that the buried range traceable in the Kalmyk-Salsk steppes is probably a plunging extension of the folds of the western slope of the Urals. Indications as to the meridional trend of the dislocations on the Chushka-Kul elevation and east of it ([7]) make us assume the existence of a branching of the Uralian system of folds shown in the scheme. As to the Mangyshlak mountains, they probably form a part of another folded structure. This mountain chain, formed by the movements of the Cymmerian cycle, stretches along the southern edge of the platform, which adjoined at that time the Hercynides of the south-western extension of the Urals.

In the Mangyshlak region typical geosynclinal forms of folding have been recorded for the Permian and the Triassic. During the Lower Jurassic the geosynclinal regime began to die away. At the Jurassic time the Mangyshlides were the northern dry land which supplied material for the formation of sediments in the

geosynclinal area situated south of them. The New-Cymmerian movements were but slightly reflected by the Mangyshlak zone, and during the Alpine orogenic process it was already a part of the platform.

One of the links of the Mesozoic mountain structure as that represented by the Mangyshlak region seems to be the Dobrudja elevation on the Danube, where the relations in the nature of the dislocations of deposits of different age are known to be analogous to those of the Mangyshlak region. During the Alpine mountain-making Dobrudja, as well as Mangyshlak, were already a part of the platform. North of Dobrudja the Mesozoic deposits are concealed again under the mantle of younger sediments. The buried extension of this structural zone may be traced in a barrier-like elevation bordering the cis-Carpathian depression from the east. Drilling on this barrier-like elevation has revealed a chain of local structures, associated with accumulations of gas in Tortonian beds in the areas of Opara, Dashava, Kalushi, *etc.*, which are the greatest in Europe.

The presence of a considerable number of oil occurrences in the Mesozoic deposits of Mangyshlak suggests a regional occurrence of oil and gas in this zone. The slight metamorphization of the rocks permits to expect the preservation of hydrocarbon compounds. West of Mangyshlak—in the cis-Caucasus and in the Crimea, *i. e.* where the Cymmerides are buried under a considerable thickness of Tertiary and post-Tertiary sediments, natural gas occurrences are likely to be present in the rocks covering the Cymmerian basement. Methane, formed as a result of decomposition of the petroleum bitumens during the sinking of the folded structure, must have migrated from strata under pressure produced by overlying strata. In connexion with this, the problem of a search for all the remaining links of the buried folded structure of Cymmerides (Mangyshlides) throughout the area between Mangyshlak in the east and the Dobrudja elevations in the west, is one of great importance in the development not only of the gas-producing, but also of the oil-producing industry of the USSR.

The map attached shows that in the zone bounding the piedmontite downwarping of the Crimean-Caucasian mountains from the north the Cymmerian basement is likely to be found throughout the area from the Tarkhankut elevation on the North Crimea to the Caspian Sea. The presence of young uplifts in this zone was recorded in the recent years by M. M. Zhukov on the divide of the Terek and Kuma Rivers at Cherny Rynok and by V. G. Loewinson, on the Eisk Peninsula. The existence of the Blagodarnensky uplift was recorded by the author as early as 1936—1937(⁸) and has been confirmed by the explorations of M. M. Zhukov. Between the Blagodarnensky and the Cherny Rynok uplift a gravity anomaly is known, viewed as a gently sloping anticline of a Caucasian trend (⁸).

While considering the above-mentioned zones of distribution of buried folded systems – the Hercynides (Uralides) and Cymmerides (Mangyshlides), it is necessary to point out that the structures of the Caucasus and the Crimea, lying to the south, should be considered, as it is seen from the map, as links of a single Alpine belt interrupted by areas of plunges across the strike of the folds.

The zones of propagation of the simultaneously uplifted folds can be traced on the basis of a zonal subdivision of the geosynclinal area, of the pre-Cambrian platform, being regularly surrounded by Caledonides, Hercynides, Cymmerides and Alpides. These zones must probably have displayed the most outspoken geoanticlinal tendencies during the period of the geosynclinal regime throughout the cycles of orogenesis. At each phase of movements the arising folded structures became welded to the platform as a result of consolidation in the marginal part of the geosyncline closely approaching the platform. At the same time in the inner, remote parts of the geosyncline, upheavals were followed by new sinking movements. The ranges formed earlier and involved in these plunges were overlain by younger sediments, with a subsequent rise during the next great paroxysm of orogenesis.

Within the Crimean-Caucasian zone intense folding movements have been recorded for the New Caledonian, Hercynian and Old Cymmerian phases, but the critical stage must have been reached during the New Cymmerian phase, for it was

Structural map of the southern border of the European part of the USSR. *1*, edge of the pre-Cambrian basement of the platform; *2*, platform elevations with supposed Caledonian folded basement; *3*, Hercynian folded structures on the surface; *4*, supposed buried extension of Hercynian folded structures; *5*—Cymmerian folded structures on the surface; *6*, supposed buried extension of the Cymmerian structures. Alpine folded structures: *7*, area composed of Mesozoic and older deposits; *8*, frontal anticlinal zones in Tertiary deposits; *9*, supposed buried uplifts; *10*, depressions of the piedmontite downwarping

not until that time that ascending movements acquired primary importance. The general sinking of the geosyncline did not compensate for them any longer. It should be noted that by this time the Crimean-Caucasian zone was the frontal part of the geosyncline, since the Mangyshlak folded zone had already been consolidated and welded to the platform.

The development of movements within the Crimean-Caucasian zone proceeded from the centre towards the periphery. During the early Palaeozoic phases were formed the folds of the present core of the Great Caucasus. During the Mesozoic time originated the folding in Mesozoic deposits, while the older folded basement was subjected to block shiftings only. During the intermediate period intense vertical shifts took place in the consolidated Mesozoic folded structure, while the Tertiary sediments were gathered into folds.

It should be borne in mind that a map of the outlines of the pre-Cambrian base of the platform surrounded by a ring of Caledonian folded structures was compiled as early as by F. N. Chernyshev ([9]).

The possibility is not excluded that future drilling will confirm the presence in the southern part of the Russian platform not only of buried Hercynides and Cymmerides, but also of zones of Caledonides traced by F. N. Chernyshev. In the map adjoined are shown the Khobda ([10]) and the Stalingrad platform arches, in the core of which folds of the Caledonian structure are probably enclosed.

The hypothetical schemes of the structure of depths in the regions covered by a thick mantle of flat-dipping sediments do not reflect the actual state unless there is a clear understanding of the fundamental relationships between the present morphological forms and the structural elements, which originated at different stages of the geologic history. All the processes which are taking place in the Earth's crust are to a greater or lesser degree reflected in the present structures of the various principal rocks, formed as a result of consecutive tectonic movements. Owing to this, the present morphological forms of the Earth's surface often show marked traces of very ancient phenomena.

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Received
8.VI.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

GEOLOGY

RESULTS OF FOSSILIZATION OF THE VIBRATORY MOTION OF THE EARTH'S CRUST AS EXPRESSED BY MEANS OF THE SERIES

$$\sum_{i=0}^{k} e^{a_i x+b_i} \cos(\omega_i x+\varphi_i)$$

By A. B. VISTELIUS

(Communicated by A. N. Kolmogoroff, Member of the Academy, 4. X. 1945)

1. In my earlier works ([2], [3]) I have shown that the distribution of porosity in vertical sections of the lower-Kazan substage in the Buguruslan region obeys the equation

$$y = m + nx + e^{a_1 x+b_1} \cos(\omega_1 x - \varphi_1) + e^{a_2 x+b_2} \cos(\omega_2 x - \varphi_2) \qquad (1)$$

where y is the porosity; x, the depth measured from the roof of the deposits under investigation. It has also been established that the values of the parameters of the periodical terms in equation (1) vary with the position of the investigated cross-section on the structural form.

After these works had been published, another 13 cross-sections of the lower-Kazan deposits exposed by drilling in the Buguruslan region were studied. The drill-holes were distributed more or less evenly over an area of about 4000 km².

As a result of this investigation it was found that approximation to a series of observations by means of function (1) is closer in the middle of the series than at its ends. In order to find out the causes of this phenomenon the differences

$$\delta(x) = y - e^{a_1 x+b_1} \cos(\omega_1 x - \varphi_1) - e^{a_2 x+b_2} \cos(\omega_2 x - \varphi_2) \qquad (2)$$

between the series of observed quantities and the periodical terms of equation (1) were investigated. The study of these differences has revealed that their representation by a linear function is only possible in the first approximation. Closer approximations are obtained by means of the equation

$$\varphi(x) = e^{ax+b} \cos(\omega x - \varphi) \qquad (3)$$

Thus, it appeared that equation (1) could be represented in the form of the series

$$y = \sum_{i=0}^{2} T_i(x) \qquad (4)$$

where

$$T(x) = e^{ax+b} \cos(\omega x - \varphi)$$

For the term $T_0(x)$ the extinction coefficient was omitted in view of the smallness of the logarithmic decrement.

In Table 1 the values of the parameters of equation (4) are given for all the drill-holes investigated.

Examining the quantities ω referred to in Table 1 it is easy to see that they fall into two groups. The first includes the values listed under ω_0, the second is formed by the values placed in the columns ω_1 and ω_2. Accordingly, two groups of terms may be distinguished in equation (4); the first is that

Table 1

Structure form	Nos. of drill-hole	Parameters of the Porosity Distribution Function											
		a_0	b_0	ω_0	φ_0	a_1	b_1	ω_1	φ_1	a_2	b_2	ω_2	φ_2
Elatma	11	+0.0238	+ 1.2030	4.5	279.0	0.0077	1.9096	21.15	64.0	+0.0475	1.0098	21.15	243.2
Sultangul	12	—	+0.8215	9.5	292.5	—	0.6436	40.00	220.8	—	—	—	—
Brenchaninov	13	—	+ 0.9833	16.8	352.2	+0.0711	0.0283	60.00	30.0	+ 0.1322	— 0.7767	72.00	288.0
Sadkin	17	—	+ 1.2019	12.3	306.1	+ 0.1254	2.7361	32.70	65.4	+0.0471	+1.2011	32.70	278.0
Piluguin	16	—	+ 1.4537	3.0	296.3	+0.0045	1.5147	25.70	13.60	+ 0.0169	+1.3876	22.50	101.2
	15	—	+1.9605	1.5	316.6	—0.1115	2.1384	25.75	326.2	—0.0236	+2.1302	24.00	150.0
	14		+1.1594	6.4	60.0	—	2.2586	25.70	171.5	—	+2.0150	20.00	160.0
Amanak	4	—	+0.9577	5.6	120.7	—0.0262	1.7790	16.29	110.08	+0.0044	+1.2580	10.11	59.2
	5	—	—	—	—	—0.0150	1.9680	16.35	197.35	+0.0260	+0.9870	14.12	110.16
Sosnovka	19	—	+ 2.0013	11.2	152.5	+ 0.0163	2.4493	21.18	264.8	+ 0.0000	+ 1.9065	42.35	348.0
	20	—	+1.6794	12.7	8.6	—	—	—	—	—	+0.8882	25.70	49.2
Kalinin-Zhuravlev	7		+ 1.9383	2.0	342.1	+0.0227	1.4280	20.00	124.0	+ 0.0287	+ 0.0900	21.82	95.96
	2	—	+0.9496	3.7	71.5	—0.0220	1.9340	19.36	66.76	+0.0220	+1.4030	19.36	108.42
	3	—	+1.3079	3.8	270.8	—0.0152	1.2410	21.42	186.36	+0.0066	+1.3980	21.42	199.25
Novo-Stepanov	1	—	+ 1.3863	5.0	351.8	+ 0.0483	2.1760	35.30	38.97	+ 0.0136	+ 0.9130	38.47	235.02
	9	—	+0.8524	4.2	39.3	—	—	—	—	—	+1.3863	30.00	73.5
	8	—	+0.5582	0.5	351.5	—	—	—	—	+0.0120	+1.2730	20.00	70.0
	6	+0.0033	+0.2430	1.5	355.5	—0.1014	3.1040	27.34	182.68	+0.0120	+0.8160	16.00	351.2
Buguruslan	10	—	+2.0630	12.7	128.8	+ 0.0391	0.2225	30.00	240.0	—	+ 0.1970	40.00	88.0
Krasny Kliuch	18	—	+1.5544	10.7	36.7	—0.0176	1.6145	18.00	306.0	—	—	—	—

with the minimum values of ω, which may be denoted by

$$\varphi(x) = T_0(x) \qquad (5)$$

and the second, with large values of ω, forms the expression

$$\psi(x) = \sum_{1}^{2} T_i(x) \qquad (5a)$$

2. Along with refining the form of the function of distribution of the coefficients of porosity, the thicknesses of the investigated deposits were studied. Their values in Table 2 are for the drill-holes in which the distribution of porosity was investigated.

Table 2

No. of drill-hole	1	2	3	4	5	6	7	8	9	10
Depth of bottom, in m	341.1	311.0	329.0	324.0	333.3	331.0	342.0	405.9	274.1	254.0
Depth of roof, in m . .	288.1	262.0	271.0	258.0	275.1	279.0	292.0	364.0	235.5	236.0
Thickness, in m	53.0	49.0	58.0	66.0	58.2	52.0	50.0	41.9	39.6	18.0

Table 2 (continued)

No. of drill-hole	11	12	13	14	15	16	17	18	19	20
Depth of bottom, in m	317.5	301.0	455.8	509.0	509.0	502.0	589.0	372.0	379.0	355.0
Depth of roof, in m . .	283.0	281.5	438.5	462.0	459.6	465.0	541.0	324.0	322.0	300.0
Thickness, in m	34.5	19.5	17.3	48.0	49.4	37.0	48.0	48.0	57.0	55.0

For the drill-holes Nos. 14, 16, 17, 18, 19 and 20, which had not been sunk to the bottom of $P_2^{kaz_1}$, the thickness was computed by means of equation (5a). This was possible because in the bottom region of the lower-Kazan deposits all drill-holes showed a minimum of the function (5a), the order of thicknesses was known and the functions (5a) were established from observations made in that part of the substage which had been exposed by drilling.

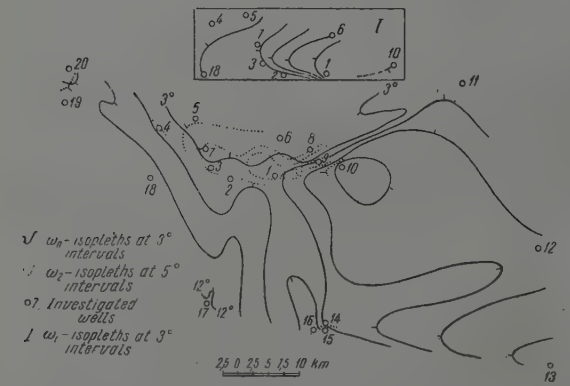

Fig. 1

3. In order to establish the relation between the parameters of the distribution function (4) and the thicknesses, isopleth and isopachyte maps were drawn (4) and compared to each other. Fig. 1 gives the position of ω_0 isopleths at 3° intervals, and that of ω_2 isopleths at 5° intervals. Scheme I gives the position of ω_1 isopleths at 3° intervals. The position of isopachytes at 7 m intervals is shown in Fig. 2. It may be seen from these schemes that there is a great similarity between the two kinds of curves as regards their position. It is even more so if we take into

account the distortion of the thicknesses caused by the superposition of vibrations of higher orders and by incidental factors of no geotectonic nature.

A closer examination of the above schemes will easily reveal that the outlines of the ω_1 isopleths and, still more so, of the ω_2 isopleths can be tied up with the local changes in thicknesses observed in the central part of the region in which the

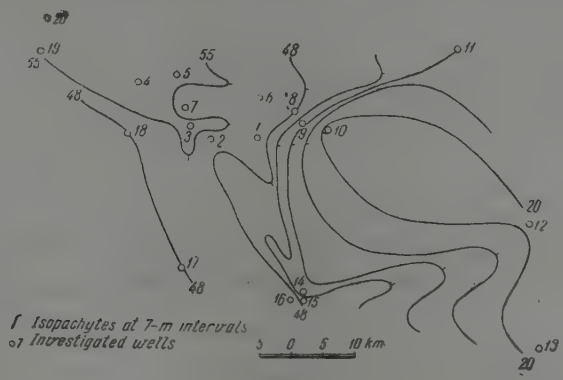

Fig. 2

drill-holes are more numerous. The values of ω will be found to be linked with the local thicknesses by a simple relation: the larger the value of ω in a particular cross-section, the lower is the thickness in this section. The same relation is observed between the ω_0 isopleths and the isopachytes which extend across the whole area of the region. Thus, we find ω_0 to be related to the general map of the distribution of thicknesses in this region. Remembering that thicknesses represent the most direct expression of the vibratory motions that took place during the period of the formation of the sediments ([1]), we are entitled to the following generalization.

The vibratory motions of the sedimentation period of the bed investigated are fossilized in the porosity distribution and may be expressed by means of the series

$$f(x) = \sum_{i=0}^{k} T_i(x), \qquad i = 0,\ 1,\ \ldots,\ k \tag{6}$$

To every order of oscillations there correspond one or several terms of the sum which are distinguished from the other terms by the value of the periodicity coefficient ω.

It is well to note in conclusion that by drawing isopleth maps for the periodicity coefficients of the porosity distribution functions we apparently can map the vibratory motions of the Earth's crust separately, according to their order. A similar method can be applied for the separation of rhythms of different order and for their separate mapping in studying any formation of rhythmic stratification. In doing this we can instead of porosity investigate the distribution of any other lithological constant characteristic of the formations concerned.

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Received 4.X.1945.

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1945. Volume XLIX, № 7

GEOLOGY

ON A DOLOMITE IN THE UPPER PALAEOZOIC OF THE URAL-VOLGA REGION

By G. I. TEODOROVICH

(Communicated by F. P. Savarensky, Member of the Academy, 8. VI. 1945)

1. While studying the carbonate deposits of the Palaeozoic (C_1, C_2, C_3, P_1) of the Ural-Volga region and the western slope of the South Urals, the author met with different types of dolomite formation. The dolomites and dolomitic limestones studied occur in the form of packets, beds, layers, thin bands and lenses; in the latter cases they usually alternate with limestones and sometimes (in a part of the sections of lower Permian lagoonal deposits) with anhydrite or gypsum. These peculiarities in the mode of occurrence are evidence of a normal sedimentary formation. As seen under the microscope, the overwhelming majority of the dolomites studied is associated with a replacement of the calcareous material of the dolomites usually with a diagenesis of the sediment, and occasionally also in the rock already shaped. The fact of the replacement of the calcareous sediment with dolomite is revealed by the presence in a great many of the dolomites investigated of relict organogenous and oölitic structures. In addition, the micrograined calcareous sediments, too, are very often replaced by coarser-grained rhombohedral dolomite; in this case a definite number of relict patches of micrograined dolomite is usually present in the rock. Primary dolomites have been encountered along with bands of replacement dolomites in typically lagoonal facies of lower Permian deposits only.

2. It has already been pointed out ([6]) that the replacement dolomite is developed selectively after micrograined $CaCO_3$, while patches of coarser-grained $CaCO_3$ are unfavourable for dolomitization. The finer-grained (taken as a whole or in some detached patches) is the original calcareous material (sediment) and the higher the mineralization of the dolomitizing waters, the smaller are the grains of replacement dolomite produced and the greater their number, other conditions being equal.

The idiomorphism of the rhombohedral grains of dolomite and their approximately equal size are characteristic of an incomplete dolomitization of the calcareous deposit, *i. e.* of the majority of dolomitic limestones. On the contrary, in the case of a replacement with dolomite of the whole or a distinctly greater part of the $CaCO_3$ of the calcareous deposit, many dolomite grains interfere with one another when growing, owing to which irregularly rhombohedral grains of about the same size are formed. In general, grains of dolomite of a rhombohedral, rounded and irregularly rhombohedral, as well as of a more complicated irregularly (rounded) polyhedral shape are characteristic of replacement; very frequently they are developed in the place of the primarily calcareous skeletal remains of

organisms (more rarely, of oölites); they are not infrequently protruding into the latter from the sides or enclose disseminated small grains of micrograined $CaCO_3$ identical with that preserved in several cement areas or in the groundmass of the rock. It should be noted that dolomite grains which have been growing during a rather long time and developing under less favourable conditions (after distinctly grained or partly recrystallized $CaCO_3$) are usually (along the edges of dolomite fields) larger and idiomorphic, and sometimes typically zonary or purer along their periphery. The grains of replacement dolomite usually possess a normal extinction, and it is only on some occasions that separate grains show a wavy extinction. The latter seems to be produced under the influence of the pressure of later mineral segregations of anhydrite, celestine, *etc.*, and less frequently after the growing dolomite grains have reached areas unfavourable to dolomitization and solidified areas.

In some cases, mostly in dolomitic limestones, the entire dolomite of the rock may belong to the same type, *e. g.* be colourless in the thin-section, rhombohedral, 0.03—0.20 mm in size. In other cases two types of dolomite grains are observed: one in the form of a fine-grained variety after skeletal remains of organisms composed of pelitomorphic $CaCO_3$, and another in the form of colourless rhombohedra 0.03—0.25 mm in size; such a picture is observed in some dolomites. In still other cases the dolomite of the earlier generations possesses less distinct boundaries, and is contaminated with a finely dispersed chemogenous admixture, while dolomites of later generations, with more distinct, usually rhombohedral outlines, are purer and more coarse-grained. Finally, in some sections the author noted the following regular relation: in dolomitic limestones the dolomite is rhombohedral, colourless in thin-section, while in normal and calcareous dolomites the grains are irregularly rhombohedral with a brownish-greyish shade in the transparent section. Such relations may be accounted for by differences in the values of rH, the influence of dry land and the beginning of dolomitization. In certain dolomites part of the grains of the groundmass of the rock, of a brownish shade in the thin-section, show a thin light-coloured peripheral fringe of dolomite. If the number of such dolomite grains in the rock is small, they are associated only with the preserved areas of distinctly granular calcite, or with the pores of the rock. In the latter case it is evident that the dolomite formed later is purer.

In the case of uniform dolomitization, the less the number of centres of growth in the course of formation—in the long run—of normal and more or less pure dolomites, the larger are the grains of dolomite, the slower their formation, and the poorer the preservation of traces of the original structure in the form of a relic structure of the rock or detached distinguishable relics. The recrystallization of dolomites to distinctly granular irregularly-isometric marbled and other varieties, frequently of twinned grains, is usually the result of a metamorphism of the rock. Such dolomites occur mostly in older deposits of folded areas. In common sedimentary dolomites the phenomena of intergrowth of adjacent rhombohedral grains and their simultaneous extinction are not observed, with the exception of a few rare cases, which is evidence against the assumption of an extensive recrystallization of dolomite.

According to the internal structure of the dolomite grains, the following types may be distinguished: (a) homogeneous; (b) with some kind of nucleus; (c) with a peripheral, usually purer fringe; (d) zonary; (e) with thin inclusions in the peripheral parts of the grain. The nucleus may consist of a dolomite grain, of ankerite (?), of micrograined carbonate (pure or in a mixture with ferruginous chlorite?), and of some other minerals. Thus, the mineral nature of the central nuclei of the dolomite grains often points to the reducing or slightly reducing conditions that prevailed during their formation. On the other hand, the presence in various parts of the dolomite grain of inclusions of micrograined calcite points to the development of the dolomite under replacement of slowly dissolving $CaCO_3$.

3. Aside from the distinctly prevalent replacement dolomite, among the grains of dolomite of the deposits studied, two other groups may be separated out on the basis of the conditions of their formation: (a) dolomite of coating of the

walls of voids in the solidified sediment or rock; (b) primary dolomite, normally granular, precipitated from the mass of water of the basin.

The dolomite of coating of the walls of voids was observed by the author in Upper Palaeozoic reefs, *e. g.* those of Ishimbay, as microaggregate grains with a cruciform extinction. These grains of dolomite show a semi-globular, semi-ellipsoidal or sectorial shape, representing microaggregates of finely fibrous grains of dolomite arranged in tufts or radially. In the same Ishimbay reefs crusted dolomite zones were occasionally observed, composed of parallel, finely fibrous individuals of dolomite, fringing the upper surface of the cavity or fissure of the solid skeleton in a continuous fringe analogous to the calcareous incrustation.

Primary, normally granular dolomite, representing a chemogenous precipitate settling from the mass of water of the basin, must usually be, by analogy with chemogenous micrograined $CaCO_3$, of very small size ($<0.005—0.003$ mm) and have an irregularly isometric, irregularly rounded or some other, generally irregular shape. The investigation of the calcareo-dolomite silts of Lake Balkhash ([4]) has confirmed these general considerations. In some occurrences in lagoonal Lower Permian deposits of rhombohedral, usually relatively large dolomite grains, amidst anhydrite, it seems probable that they consist of a small nucleus of primary dolomite, which has not preserved its outlines, and while growing, assumed idiomorphic outlines and an increased volume, already on the bottom of the basin.

4. The predominance of replacement dolomite among marine and lagoonal deposits is a common phenomenon ([8], [3], [10], [7], [5], [6]), associated with the salt composition of the water of the corresponding basins. Recent studies have furnished ever increasing evidence to the effect that the formation of dolomite proceeds in seas and most of the lagoons through the change of the originally calcareous sediment in interaction with the near-bottom and silt water ([9], [1]), and sometimes through the drying up of the calcareous sediments. The primary dolomites may originate under conditions of lagoons salinified to the stage of $CaSO_4$ precipitation (or approaching the latter), mostly if they are supplied from outside with waters containing dissolved $CaCO_3$. In this case mainly primary spotted dolomite-sulphate rocks are formed. Primary chemogenous calcareo-dolomite sediments are possible rather in basins with water of the hydrocarbonate type, upon their evaporation under conditions of arid climate; such are the silts of the eastern parts of Lake Balkhash and its shallow-water bays, where the concentrations of salts are the highest of those recorded in the lake ([2]). Among the fossil dolomites of this type, lakes are very rare.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

MINERALOGY

ON THE FORMATION OF AGATE OÖLITES

By A. M. KUZNETZOV

(Communicated by D. S. Beliankin, Member of the Academy, 18. VII. 1945)

The sulphate-calcium formation of the Kungurian in the near-Ural area contains silica formations in a disperse state and as: 1) a reticulate framework with a gipsum filling; 2) lenticular concretions in a paragenesis with anhydrite; 3) small grains of flint in gypsum, and 4) oölite accumulations. The present paper deals with these latter, with the object of explaining the origin of oölites of an agate structure on a specimen of oölite, found in an exposure of the anhydrite-gypsum formation on the left bank of the Kama, 6 km upstream of the point of discharge of the Chusovaya river into the Kama.

The formation referred to above consists of closely packed globular and ellipsoidal oölites, solidly cemented together by calcium and magnesium carbonates and chalcedony, with an admixture of iron and manganese compounds and gypsum. According to the analysis and determination in thin-sections the cement makes up 30 to 35 per cent. The oölites can be separated with acid and by mechanical stress; it appears then that a considerable part of the oölites bear traces of disturbances, deformation of their rounded shape, revealed as faintly pronounced flat faces at the points of contact of the oölites with one another. The oölites are dark-grey, with light spheres in the centre; many have cracks of the type of dehiscence lines of spores. They differ little in size and weight. The average weight of an oölite, from 200 weighings, is 11.2 mg, diameter 2.03 mm, specific gravity, 2.592.

The substance of the oölite is characterized by the following composition in per cent: SiO_2, 91.16; $Al_2O_3 + Fe_2O_3$, 2.60; CaO, 1.92; MgO, 0.65; SO_3, 0.68; MnO, 0.18; H_2O at 110°, 0.24; loss on ignition, 2.04. The high value of the loss on ignition is probably accounted for by an insufficient removal of carbonate, the presence of hydrate water in the Fe and Mn compounds, of crystallization water of the gypsum and a possible admixture of organic matter.

The oölites have a very distinct zonary structure (see figure), seen under the microscope in thin-sections, the following being distinguishable: 1) a dark-yellow nucleus, 0.2—0.3 mm in diameter; 2) an enveloping zone of yellow colour, 0.2—0.3 mm thick, and 3) an outward grey zone 0.4—0.5 mm thick. The two zones consist of concentric alternating pale and dark rings 5—20 microns wide, visible in ordinary and polarized light. The central part of the oölites is only partly embraced by the zonary structure. In thin-sections radial microcracks up to 0.1 mm wide are visible, not infrequently cutting the whole of the oölite, some of them being filled with gypsum and a dark-brown substance with breaks in it.

The oölites represent chalcedony spheric crystals, built up of biaxial positive and uniaxial negative, radially oriented fibres, giving on the whole a distinct picture of a dark cross with the nicols crossed. Some spheric crystals in concentric bands have a variable optical sign, or else it varies according to sectors disjoined by cracks. The interference colour alternates in dark and light rings and rarely varies within the rings themselves*. The dark bands are distinctly seen with an extreme magnification of 50, and appear diffuse at 100 and an extreme magnification of 200. They have a sharply delimited edge from the side of the nucleus,

* The determination was made by A. A. Aprodova.

but their colour is gradually fading towards the other, outward edge, passing to a diffuse one. The number of dark periodic rings varies from 30 to 50. Not infrequently in whole series of bands microshifts are seen along the cracks.

From the analysis it is seen that more than 0.9 of the substance of the oölite is made up of SiO_2, the content of which will be higher still if the substance filling the cracks be excluded from the composition, but this could not be determined. Typically enough, the accumulation of manganese in them is high, and in the mass of the cement reaches 0.55 per cent. This should be taken to mean that the pH of the medium was low at the time of manganese accumulation.

The graded condition [1] of the oölites is evidence of the fact that they have arisen in a suspended state. Originally, the centres of coagulation of SiO_2, supplied to the saline basin by waters from the dry land, seem to have been positively charged suspended particles of calcium carbonate [2]. Sinking to the bottom the nuclei thus formed increased their mass by orthokinetic coagulation, assuming a round shape and about the same size.

While settling, the aggregates became closely packed and compressed, losing their rounded shape. At the same time, as the aggregates were reaching the near-bottom region, components of the near-bottom solution were diffusing to the mass of their silica gel [3], soluble compounds of iron and manganese, apparently their carbonates, precipitated rhythmically as zonary coloured bands, possibly in the form of sulphides, since adsorbed hydrogen sulphide in the silica gel might also be present. The possibility is not excluded that rhythmically precipitated sols of iron and manganese were also diffusing into the porous silica gel. The lower pH value of the silica gel, the negative charge of the sol of manganese and the positive charge of the sol of iron could cause their penetration to the gel to a different depth, owing to the fact that the manganese could migrate more freely in the negatively charged silica gel than iron did. The outward zone of the oölites was formed under precipitation of iron, the inner zone arose under precipitation of manganese, in the form of coloured Liesegang rings separated by zones free from the precipitate.

In the course of time the gel of the aggregates was ageing, crystallized, suffered compression, cracks appeared in its mass, filled up with the same components of the medium which coagulated with breaks of the brown mass within the cracks themselves. The non-uniformity of compression caused the appearance of microshifts in the mass of the silica gel, which is evidenced by a shifting of series of bands. The substance seems to have crystallized at a different rate in different parts of the aggregates, which accounts for a certain optical difference mentioned above.

The study of silification phenomena in the sulphate formation of the Kungurian, along with the study of its other properties, will contribute to a better understanding of the conditions of formation of the thick [4] series of sediments of calcium sulphate, which in many respects still remains a puzzling problem. The formation itself is an excellent example of the ageing of a mineral gel with rhythmical sediments, formed in a water resevoir.

Received
18. VII.1945.

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1945. Volume XLIX, № 7

PLANT PHYSIOLOGY

EFFECT OF IODINE UPON THE ACTIVITY OF TOBACCO MOSAIC VIRUS

By P. AGATOV

(Communicated by A. A. Richter, Member of the Academy, 4. VIII. 1945)

The effect of chemical agents upon the activity of the various viruses has been studied by a number of workers ([1-11]). The objects of their studies were different. Some of them attempted to find substances able to reduce or even to supress completely the activity of the virus in order to approach more closely the study of the nature of the virus itself, or to control virus diseases. In other studies attempts were made to discover substances which would make it possible to isolate the virus in a more or less pure form, leaving the activity of the virus intact. Still others were devoted to a search for substances affecting different viruses in a different way, which might serve as one of the taxonomic characters and might be helpful in isolating definite viruses from a mixture.

The present author arranged experiments on the effect of certain reagents possessing different reduction-oxidation potentials upon the activity of the tobacco mosaic virus. Among many substances applied by me for this purpose, special attention was given to iodine.

It was known from literary data ([7]) that the treatment with a 0.1 per cent iodine solution with a pH 6.8 during three days does but slightly affect the activity of the mosaic virus. It was known also ([11]) that oxidation of the sulphhydril-groups of the virus protein by iodine does not reduce the activity of the latter.

In the course of my studies on the effect of iodine with pH values varying within a wider range, I became aware of a varying effect of iodine upon the activity of the tobacco mosaic virus. The following experiment was conducted in order to clear up this point: 1 ml of the 0.15 *M* of the phosphate buffer of a corresponding pH, and 0.5 ml of a 0.2 per cent solution of iodine in potassium iodide were added to 1 ml of an aqueous solution containing 1 mg of the virus (prepared by the method of Rischkov and Gromyko ([12])). The treatment lasted 4 or 24 hours; after the treatment the corresponding products were subjected to dialysis, and their activity was determined by the leaf-halves method on the leaves of *Nicotiana glutinosa*. As a control served an aqueous solution of the tobacco virus mosaic. The results of these experiments are presented in Table 1.

From these data it may be seen that with a pH equal to 5.5 - 6.0, iodine had either no influence at all, or had but a slight effect upon the activity of the virus in question. When the medium had a more acid or a more alkaline reaction, iodine caused a considerable reduction in the activity of the tobacco mosaic virus, which attained 100 per cent with a pH = 4.5 and 8.0.

The results obtained did naturally call for a quantitative study of the interaction between iodine and the protein.

For recording the amount of iodine which entered reaction with the protein, I took 20 ml of an aqueous solution containing 10 mg of the virus; it was added 20 ml of a 0.15 *M* phosphate buffer possessing a corresponding pH value, and 10 ml

Table 1

Activity of the Virus after Treatment with Iodine, Expressed in per cent to its Initial Activity

pH	Duration of treatment		pH	Duration of treatment	
	4 hours	24 hours		4 hours	24 hours
4.50	15	0	6.98	31	12
4.94	15	8	7.38	12	0
5.59	92	71	8.04	4	0
5.91	133	71	8.34	0	0
6.43	74	6			

of a 0.01 *N* aqueous solution of iodine with potassium iodide. After 24 hours the mixture was acidified with acetic acid, and the excess of iodine which had not entered reaction was titrated with 0.01 *N* hyposulphite solution. Data on the quantity of iodine which entered into reaction with the protein of the virus are contained in Table 2, column 2.

Table 2

pH	Amount of iodine in mgs per 1 mg of tobacco mosaic virus		
	entered into reaction	associated	spent on oxidation
1	2	3	4
4.5	0.0367	0.0343	0.0024
5.9	0.0726	0.0211	0.0515
8.3	0.3691	0.0289	0.3402

The experimental conditions were the same as in the preceding experiment. After a 24-hour-long action of the iodine upon the protein of the virus, the latter was isolated by dialysis, the solution of the protein was evaporated, and the dry substance was subjected to cindering by melting it with KNO_3 and K_2CO_3. The resulting product was dissolved in sulphuric acid, and the iodine contained in the potassium iodate produced was quantitatively determined. The corresponding data are contained in Table 2, column 3.

By subtracting the amounts of iodine presented in Table 2, column 3, from the corresponding amounts in column 2, we obtain the amount of iodine left undetermined in titrating; this iodine had evidently reacted with the virus protein as the oxidizing agent. The corresponding data are contained in Table 2, column 4.

It may be seen from Table 2, column 3, that, according to the different pH values different amounts of iodine become associated with the virus protein. The minimum iodine amount is bound at a pH=5.9; at this pH value the iodine entering the protein molecule of the tobacco mosaic virus does not appreciably affect its activity (Table 1). Inactivation of the virus by iodine at pH values higher than 5.9, probably takes place at the expense of the oxidizing action of iodine upon the active protein groups, the amount of iodine spent on protein oxidation with a pH=8.3

being greater than at a pH =5.9. Inactivation of the virus at pH values below 5.9 probably takes place through the association of iodine with the active protein groups of the virus, possibly at the place of the double bond existing in them, which undergo tautomeric re-arrangement at different pH values. This suggestion seems to be plausible from the data of Table 2, columns 3 and 4, which show that at a pH =4.5 the amount of iodine spent on oxidation of the protein of tobacco mosaic virus is only a small fraction of that at a pH =5.9, when the tobacco mosaic virus remains active. As to the amount of iodine associated with the protein of the tobacco mosaic virus, it is larger at a pH 4.5 than at a pH =5.9.

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4.VIII.1945.

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1945, Volume XLIX, № 7

PLANT PHYSIOLOGY

EFFECT OF LIGHT UPON PIGMENT FORMATION IN *ASPERGILLUS FLAVUS*

By V. E. PONTOVICH

(Communicated by A. A. Richter, Member of the Academy, 28. V. 1945)

It follows from a number of studies that light acting upon fungi and bacteria does not affect the formation of red pigments ([1,2]), whereas the formation of yellow pigments is strongly stimulated by it ([1,3-6]). Studies on the influence of the natural and artificial filtered light have likewise shown that blue light only (wave-length from 430 to 485 mμ) produces stimulating effect upon pigment formation, while red light has no influence upon it ([4,6-9]).

It is a familiar fact, too, that light (especially blue light) stimulates the formation of carotine ([8, 9, 10]). We have shown on a previous occasion that the yellow pigment produced by *Aspergillus flavus* was flavine ([11]). The intensity of the formation of this pigment (as well as of some others) depends on the conditions under which the fungus is cultivated ([2, 5, 12]). Since the question on the formation of flavine (vitamin B_2) by microorganisms is of interest both from theoretical and practical point of view, we have found it worth while to arrange experiments on the production of flavine by *Aspergillus flavus* under varying conditions. From the results of the investigation referred to above ([8, 9, 10]) one might expect that light would in this case, too, be the agent responsible for the stimulation of flavine formation. For this reason experiments were arranged on the effect of artificial light on *Aspergillus flavus*. In contrast to the experiments described in the papers cited above, where the bacterial or fungal cultures were illuminated for a short period of time, we have subjected *Aspergillus flavus* to continuous illumination throughout the period of development. When illumination lasts for a long period and is continuous, variations in the intensity of the acting light have a slighter effect than with short exposures. The value of our experiments is therefore not impaired by the fact that no data on light intensity were available. The experimental scheme is explained in the figure.

A light beam (from an 150 watt incandescent lamp) reached the fungus growth from below, which ensured direct illumination. The area of the glass *r* was sufficient for placing two Vinogradsky flasks of 500 cm³ capacity each; 100 cm³ of the nutrient solution was poured into each flask, the depth of the liquid not exceeding 1.5 cm. Modified Knopp's nutrient medium was employed; it consisted of two solutions (*a* and *b*, see below), mixed at a ratio 5 : 1 after being sterilized separately. The medium had the following composition:

Solution *a*: $Ca(NO_3)_2$, 0.6 g; KNO_3, 0.15 g; $MgSO_4$, 0.15 g; $Fe_2(SO_4)_3$, 0.006 g; aq. dest., 500 cm³

Solution *b*: $KH_2PO_4 + K_2HPO_4$ 1/1 (1/3 Mol.), 0.15 g per 100 cm³

When the two liquids were sterilized separately, the resulting medium was quite transparent, which was necessary for the experiments. Organic component

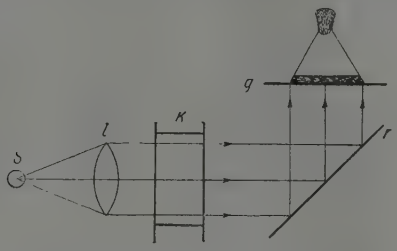
Development of *Aspergillus flavus* in the Dark and Illuminated with Filtered or Unfiltered Light
(Values referred to dry weight, without ash)

Illumination conditions	Temperature °C	Weight of yield, mg	Coefficient of utilization substance, %	Specific heat of combustion, gkal/g	Coefficient of energy utilization, %	Amount of spores, %	Amount of glucose given gkal	Note
White light (unfiltered)	17—18	449.6	31.36	5522.2	51.40	14.11	6647.5	Pigment formation begins on the 3rd day. The pigment is of intense colour. Medium, bright-yellow
»	17—18	525.1	32.09	5692.5	52.54	18.10	»	
Dark*	17—18	581.1	32.71	5262.7	49.55	45.49	»	Pigment formation begins on the 5th day. The pigment is a dull yellow. Medium, light yellow
Dark*	27	562.9	31.69	5033.7	45.91	46.75	»	The pigment is a slightly brighter than in foregoing experiment
Blue light	18—19	441.2	28.49	5671.0	47.10	22.59	7255.3	Pigment formation on the 3rd day. The colour of the pigment is bright. Medium, intense yellow
»	18—19	466.3	29.82	5662.4	48.68	20.02	»	
Dark	18—19	508.7	30.52	5405.4	48.60	40.02	»	Pigment formation on the 6th day. Medium, colourless
Red light*	19—20	591.3	32.27	5403.6	51.02	41.90	6857.3	Pigment formation on the 5th day. Pigment of a rather intense colour. Medium, colourless
Dark	19—20	560.4	30.59	5317.2	48.50	46.38	»	Same as in foregoing experiment

* Average values for two cultures are presented.

was introduced in the form of glucose in the quantity of 2 per cent. Experiments were arranged both with filtered and unfiltered light. In the latter case a cuvette *k* served as a screen; it was filled with a 20 per cent solution of copper sulphate, or with a saturated solution of potassium bichromate. The experiments were carried out at room temperature which fluctuated from 17 to 20° C. The first experiment on unfiltered light was supplemented by two control experiments, *viz.* at room temperature and at the optimal temperature (27°C).

The rate of development of the cultures differed as to the experiment; the development rate of the cultures was determined from the amount of glucose consumed. The experiment was interrupted as soon as the glucose had disappeared



S, light source; *l*, condenser; *k*, cuvette; *r*, mirror; *g*, glass table for cultures

from the medium. The following criteria of the normal development of the fungus were employed: 1) the weight of the mycelium formed (yield); 2) normal spore production (colour and proportion of the spores, determined by making preparation of the fungus film ([13])); 3) normal pigment formation (intensity of the coloration of the fungus film and of the liquid).

We were interested, besides, in the degree to which light had affected the exchange. If metabolism is affected by light absorption and the disturbance is a considerable one, a modification of the coefficient of utilization substance and of the coefficient of energy utilization must take place ([14, 15]). We determined, therefore, in our experiments not only the indices enumerated above, but the values of these coefficients, too. The results of the experiments summed up in the table support the suggestion as to the effect of light upon the formation of flavine. It appeared that the blue light absorbed by the pigment had a stimulating effect upon its production, while the red light which was not absorbed by the pigment, had no effect upon the intensity of its formation.

The character of the development of *Aspergillus flavus* under the influence of unfiltered light, or of blue light differs from that in cultures grown in the dark, or under red light.

With unfiltered or with blue light there was observed an earlier and a more abundant formation of the pigment. At the beginning of the experiment the mycelium had a bright yellow colour, while towards the end it assumed a deep-orange, or a nearly brown tint. The nutrient medium assumed also a bright yellow coloration, while under normal conditions it remained colourless, or was but slightly coloured.

Along with an abundant pigment formation in the fungus, there was also observed a weaker growth, which was expressed in a lowered yield (see table). It was of interest that the relative amount of spores was reduced (from 45.5 to 16.1 per cent), and their coloration had changed from a bright green to a light grey one. A similar phenomenon was observed by others writers ([6]).

It should be noted that there was also increase in the specific heat of combustion of the mycelium (200 gcal/g, on the average). As shown by direct determinations, the specific heat of combustion mainly relates to vegetative cells of the mycelium rather than to spores. This could be due to the accumulation in the fungus of certain substances with a higher specific heat of combustion as compared

with that of the carbohydrates. The same could be responsible for the increased coefficient of the energy utilization observed in these cultures. The value of the coefficient of the utilization of substance does not differ from the corresponding figures in the control.

When illuminated with red light, the development of *Aspergillus flavus* did not differ from that in the dark with the only difference that a slight increment in the coefficient of energy utilization was observed (see table).

To sum up, one may say that under the influence of light absorbed by the flavine pigment of *Aspergillus flavus* there takes place an intensified formation of flavine, as well as the accumulation in the body of the fungus of certain substances possessing a higher specific heat of combustion than that of carbohydrates; the same may probably account for the increased coefficient of energy utilization. An inhibition of spore formation and a modification of the colour of the spores were also observed. Red light which is not absorbed by the pigment had no effect upon the development of the fungus.

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Received
28.V.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

EVOLUTIONARY MORPHOLOGY

ON ONTOGENETIC AND PHYLOGENETIC RELATIONS BETWEEN THE SKELETON-FORMING AND GLANDULAR FUNCTIONS OF SKIN IN FISH

By B. S. MATVEYEV

(Communicated by I. I. Schmalhausen, Member of the Academy, 16. I. 1945)

The skin as a typical ectosomatic system of organs, endowed with highly diversified functions, is a very convenient material for investigating the adaptive changes of form and function in their interrelation with the environment. The study of the adaptive changes of the structure and function of the skin at different stages of ontogenesis enables the experimenters to arrive at rather plausible conclusions as to its evolutionary changes during phylogenetic development. It should be pointed out, however, that in a large part of experimental researches the skin is not considered as an entire system of organs. Nearly in all reviews on comparative anatomy and comparative histology, as well as in textbooks ([2],[3],[5],[6],[9–15]), nothing to say of separate papers, the study of the skin is subdivided into several parts isolated from one another, *viz.* research on epidermis, corium, cutaneous glands, horny, skeletal formations. The ontogenetic and phylogenetic relations existing between these constituents of the skin as an entire system of organs have been but little investigated. At this juncture I undertook a comparative embryological investigation of the structure of the fish skin at different stages of ontogenesis. The purpose of this investigation was to make clear the following questions: 1) the scale development as influenced by changes in the relation between the epidermis and corium in the embryonic development, and the rôle of these changes in the evolutionary transformation of scales in phylogenesis; 2) the relation between the skeleton-forming and glandular function of the skin in ontogenesis and phylogenesis.

The question as to the rôle played by the changes that take place during development and in the degree of differentiation of the scale rudiments in ontogenesis in their transformation in the course of evolution was the first to be solved. The investigation of the influence of changes in the relations between the elements constituting a morphogenic system, *i. e.* of the shifts in the morphogenetic relations between two rudiments, one of which plays the part of an activator, while the other is a reactor, is the subject of developmental mechanics. The investigation of the mutual interaction of the parts of these morphogenetic systems ([17]), in the ontogenesis, or form-building apparatus ([16]), as they are called after Filatov, can well be approached on an experimental way. The real nature of the formative influence of the hormone factors of the one rudiment upon another cannot be made out unless by way of experimental research. On the other hand, the comparative embryological investigation is the only way to inquire into the phylogenetic development.

Reliable information as to the factors responsible for the changes that take

place in the scale morphogenesis in the course of evolution is supplied by comparative embryological investigation of the development of fish scales (*), even without experimentation. I have pointed out more that once that during embryonic development, when scale rudiments are formed between the epidermis (which is of ectodermal origin) and the corium (which is of mesodermal origin), there arises a mutual interaction of the type described above. The epidermal enamel organ, as an initial rudiment of the scale, plays the part of an activator of the source of formative influence, while the mesodermal nipple acts as a reactor, perceiving the influence of the enamel organ. The character of skeleton formation in the mesodermal nipple depends upon the degree of development of the enamel organ; it is therefore possible to speak of an influence of the morphogenic factors produced by the epidermis. The process of dentine formation is a primary one; it is always accompanied by the development of a vigorous enamel organ,which takes place in the development of placoid scales in dog-fish. The underdevelopment of the enamel organ found by me in the skin of sturgeon fish (*) is responsible for the failure of dentine formation.

The form of the scales of sturgeon fish is similar to that of placoid scales (plates with a tooth), but they have neither dentine nor enamel. The process of skeleton formation consists in that the scleroblasts are embedded into the osteoid tissue, producing a true bone with osseous cells. A further underdevelopment of the enamel organ, which in a rudimentary form was found by me in *Teleostei* at the earliest stages of development, leads to further changes of the process of skeleton formation. An interaction between the epidermis and the corium takes place during this process. The development of the enamel organ does not begin until exfoliation of epidermis from the corium takes place. As soon as the rudiment of the enamel organ in the form of a swelling of the basal layer of the epidermis has arisen, a concentration of scleroblasts beneath the rudiment takes place, leading to the formation of a thin plate of osteoid tissue, deprived of osseous bodies, which ie characteristic of cycloid and ctenoid scales.

In fish with ganoid scales (*Polypterus, Lepidosteus*), as well as in fossil *Palaeoniscidae* and *Crossopterygii*, the process is complicated by the presence of several generations of scales. Through the fusion of the latter and the transformation of separate dentine cones are formed the characteristic structures of these compound scales (*). Comparative embryological investigations of the skin development in fish show that the degree of development of the enamel organ depends upon the degree of development of the epidermis and its differentiation.

The enamel organ begins to differentiate in the basal layer of epidermis only after the epidermis has reached a definite degree of differentiation, which depends in the first place upon the developmental stage of the embryo. The character of scale morphogenesis is also dependent on the degree of skin differentiation. However the degree of differentiation of the epidermis depends not only upon the stage of development. A number of other agents are operative in this process. For example, the sequence in the distribution of scale development depends upon the degree of differentiation of the skin over the different parts of the body. It is of interest to note the difference in the degree of differentiation of the epidermis as dependent upon the external environment of fries. For example, the young in *Trygon pastinaca* develop in the uterus of the mother up to a complete transformation into young skates. Consequently, their epidermis keeps the embryonic features and consists of two rows of cells forming a very fine layer. As a result, the scales fail to develop altogether. In oviparous skates and dog-fish the embryos hatch from eggs much earlier and, accordingly, their skin develops large placoid scales whose function is to protect the animal. Another example is furnished by two representatives of the *Diptychus* genus. One of these species—*D. dybowsky* (according to information obtained from N. N. Disler) is living in running mountainous rivers. The young of this species are characterized by strongly developed epidermis and cycloid scales in the skin. The young of *D. maculatus* are living in still water, their epidermis is poorly developed and the skin is naked.

On the basis of the facts described we have every reason to conclude that the

differences in the structure of the scales in various fish are the result of the changes of skeleton-forming function of the skin, due to the different intensity of the action of the epidermal enamel organ upon the skeletogenic mesenchyme. We are facing here an example of evolution of the form-building apparatus, the trend of which is determined by the relation between the skin and the environment at different stages of ontogenesis.

Along with its ability to induce skeleton formation, the epidermis is also characterized by its glandular function. Comparative embryological investigations of the skin development in fish reveal the antagonistic nature of these two functions. The skeleton-forming function depends upon the intensity of the glandular function: the stronger the former, the weaker is the latter. The glandular function replaces the skeleton-forming one. We have to do with a example of a phylogenetic change of an organ, which, after Sewertzov, is called an function substitution. The skin skeleton and the mucific function are playing the same rôle, protecting the organism from external influences, but this function is realized in different way, being dependent upon different conditions. When the glandular function of epithelium is hypertrophied, the skeleton-forming function may be completely suppressed.

Indeed, forms in which a large amount of mucilage is produced, are naked and deprived of skeletal elements. The following examples may serve to illustrate this relation. *Ostracodermi* representatives among *Agnatha* have a developed skin skeleton, while *Cyclostomata* representatives are naked. In *Elasmobranchii* belonging to *Chondrichthyes* the skin is covered with scales, while in *Holocephala* representatives the skin is naked. Various mobile forms belonging to *Osteichithyes* and living on hard bottom have a well-developed skin skeleton, while the skin of sluggish forms, living on the slimy ground, is naked and covered with a large amount of mucilage. Platé [19] points out that in *Torpedo* the skin is rich in mucific cells and deprived of scales. In the adult dog-fish the mucific cells are present near the fin-spines only.

Among *Teleostei* species bulbous serous glands are present only in *Physostomi* (with the exception of *Salmonidae*). In *Physoclisti* they are nearly missing.

The relation here discussed becomes especially conspicuous if we compare skin development in catfish. As well known, *Callichtys* [4,7] possesses a developed osseous armour at the external surface of which are dentine teeth homologous to the placode scales. In *Silurus glanis* the skin is naked. In *Callichtys* the osseous armour is formed through the fusion of scales of two generations. In fries 19 mm in length, when their epidermis consists of 3 layers of cells only, with very rare glandular cells, a rudimentary enamel organ is formed which looks like a large tooth. Beneath the latter a band of scleroblasts is formed, which make up a bony plate which later fuses with the tooth. This is the way in which the osseous shields are developed in the skin. In fries 29—33 mm long, having a stratified epidermis with rather rare large serous glandular cells (Leidig's and granular cells), true dentine placoid scales [12] are fusing to a long armour. In fries of *Silurus glanis*, when about 80 mm in length, the skin is naked, the epidermis is stratified, with a large number of very large serous cells, filled with transparent secretum. The number of secreting cells varies as to the part of the body. In the abdominal region the skin is thick and serous bulbous cells form 2—3 layers. In the caudal region the skin is by one third thinner. The large serous cells in the stratified epidermis are arranged in 1—2 rows, since the caudal part of the body fulfils the locomotive function and the glands present here are less numerous. In the skin covering the fins the epidermis, though stratified, contains very few glandular cells, but beneath the epidermis scleroblasts are developed and formation of osseous plates of finny radii takes place. Therefore, the skin skeleton is not developed in those parts of the body where the glandular function of epidermis is strongly developed, while in those parts of the same organism where the glands are poorly developed, the skeleton-forming function is preserved. It is of interest to note that the skeleton-forming function is preserved in latent condition for a very long time. Thus, in a catfish one-year-old, 150 mm in length, on the operculum scale-like formations were found which proved to

be rudiments of dentine cones covered with an epidermal case corresponding to the enamel organ of the placoid scale.

The above data make us conclude that the mucific function developed instead of the skeleton-forming function is the agent responsible for the reduction of the skin skeleton in naked fishes and amphibians. In this way the problem as to the structure of the skin in the ancestors of *Cyclostomata* seems to receive a plausible interpretation. The abundant secretion of mucilage by the skin of recent *Cyclostomata*, as well as the presence of a skin skeleton in all *Ostracodermi*, give us right to conclude that the ancestors of recent *Cyclostomata*, like ancient *Ostracodermi*, possessed placoid scales which they lost because of a powerful development of the glandular function of the epidermis, which substituted the skeleton-forming function of their ancestors.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 7

PARASITOLOGY

ENTOCOLAX RIMSKY-KORSAKOVI NOV. SP., A NEW MOLLUSK PARASITIC OF *MYRIOTROCHUS MITSUKURI*

By A. W. IWANOW

(Communicated by K. I. Skrjabin, Member of the Academy, 19. VI. 1945)

While working in 1931 and 1932 in the Far East I discovered in the body cavity of a Holothurian *Myriotrochus mitsukuri* from the Sea of Japan several specimens of worm-like parasitic *Gastropoda*. The parasite proved to be a new species of the genus *Entocolax* (*Entoconchidae*). Each infested fish was found to contain a single female mollusc, so that I had at my disposal a total of three female specimens. One of them was found to contain in its brood chamber, alongside of cocoons with embryos, also several dwarf males.

The females were attached to the intestine of their hosts by their posterior end (siphon), while the anterior end of the mollusk lay free in the coelom of the fish. An analogous mode of fixation is typical of *Entocolax schwanwitschi* Heding, occurring in the Kola Bay ([5]). Unlike this, other species of *Entocolax* (*E. ludwigi* Voigt, *E. schiemenzi* Voigt and *E. trochodotae* Heding) attach themselves to the body-wall of their hosts by means of their siphons.

The habitus of the *Entocolax* here described is typical of this genus. The anterior end of the body, corresponding to the proboscis of *Gastropoda* and containing the alimentary organs, was found to be similar in all specimens: it becomes gradually narrower towards the anterior end and is completed by an anterior lobe, which is distinctly separated by means of a circular furrow (Fig. 1). On this lobe is terminally located the oral opening (*o*). In young individuals the posterior part of the body, formed by the pseudopallium, also becomes gradually narrower posteriorly (*ps*), whereas in mature females it is swollen in the form of a vesicle, owing to the numerous cocoons overfilling their brood chamber. The body is completed posteriorly by a small cylindrical siphon (*si*). The latter is sharply delimited from the brood chamber and is not subdivided into several parts differing as to their structure, as it is the case in *Entocolax schwanwitschi*.

The body length of the largest female attains 14.2 mm. The whole body part anterior to the visceral sac (Fig. 1, *sa. v*) is occupied by alimentary organs. The mouth leads into a sharply delimited rudimentary oesophagus (*oe*). The whole space surrounding it is filled up with connective tissue and muscular elements, *i. e.* the circum-oesophageal sinus (rudiment of the primary body cavity) is absent. The oesophagus opens into the intestinal sac lying behind it through a very short and narrow connective tube. The latter is surrounded by small, multicellular glandular saccules, oesophageal glands, which open into the tube (*gl. oe*). The voluminous intestinal sac is blind posteriorly. Its walls are made of folded hepatic epithelium, with typical inclusions in the cells. A peculiar feature of the intestinal sac is the very strong development of the

circular folds which penetrate deep into the cavity, and the subdivision of the posterior part of the sac into well developed lobes (*lb. i*). Owing to this, the intestinal sac of the *Entocolax* here described has a shape more complicate than that of other species.

The visceral sac is better developed than in the rest of species (*sa. v*). Its largest part is occupied by the ovary. The latter consists of a short main trunk and several poorly developed branches (*ov*). Like in the rest of *Entoconchidae*, the gonoduct forms a loop-like curvature. Its proximal part represents the oviduct (*ovd*), while the distal one is differentiated into a thick-walled glandular uterus (*ut*). Connected with the oviduct is a small saccular receptaculum seminis (*rc*). The gonopore of the female lies at the surface of the visceral sac (*o* ♀).

Rudiments of the nervous ganglia seem to be absent. The sense organs are represented by a single statocyst (*st*) which is situated at the side opposite to that of the genital opening, as in *Entocolax schwanwitschi*, but lies much closer to the opening, at a distance of about $^1/_4$ of the circumference.

The pseudopallium begins from the base of the visceral sac, and is divided into a brood chamber and a siphon. The former consists of a thin-walled sac, where the dwarf males are located and where the development of the larvae takes place (♂, *coc*). The latter

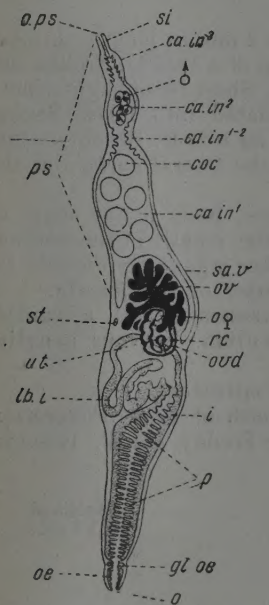

Fig. 1. *Entocolax rimsky-korsakovi* n. sp. ♀. *ca. in*¹—*ca. in*³, parts of the brood chamber; *coc*, cocoon; *gl. oe*, oesophageal gland; *i*, intestinal sac; *lb. i*, lobes of the intestinal sac; *o*, oral opening; *o.e*, oesophagus; *o.ps*, siphonal opening; *ov*, ovary; *ovd*, oviduct; *o* ♀, female genital opening; *pl t*, folds of intestinal sac; *ps*, pseudopallium; *rc*, receptaculum seminis; *sa. v*, visceral sac; *si*, siphon; *st*, statocyst; *ut*, uterus; ♂, males

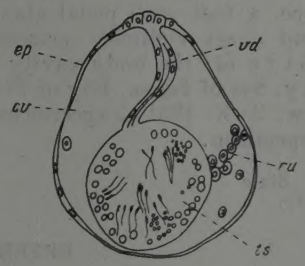

Fig. 2. *Entocolax rimsky-korsakovi* n. sp. ♂. *c*, body cavity; *ep*, body wall; *ru*, rudiments of intestine and other organs; *ts*, testicule; *vd*, seminal duct

represents a well-differentiated posterior part of the pseudopallium; it is thick-walled and perforated by a narrow canal provided with ciliae (*st*). The parasite is fixed to the intestinal wall of *Myriotrochus* with the aid of its siphon; through its canal the male larvae probably penetrate into the brood chamber where they complete their developmental cycle.

The male of the *Entocolax* here described has been found to be as strongly reduced as the dwarf males of *Entocolax schwanwitschi* and *E. ludwigi* ([5],[2]). From these it is distinguished by a higher degree of development of its seminal duct (Fig. 2, *vd*).

Thus, the following distinctive features make it possible to differentiate the species under description from any other *Entocolax* species: 1) the development of special oesophageal gland; 2) the lobate structure of the intestinal sac; 3) the situation of the female genital opening at the surface of the visceral sac; 4) the

presence of a single statocyst; 5) the simple undifferentiated siphon; 6) the more pronounced development of the seminal duct in dwarf males.

Such a form of oesophageal glands is encountered neither in *Entoconchidae* nor *Stiliferidae* and is, as a rule, not typical of *Prosobranchia*. The shape of the intestinal sac and the arrangement of the female genital opening, as well as the simple structure of the siphon, are primitive characters.

On the basis of all the peculiarities listed above the species under description should be ranked as an independent species, for which the present author has suggested the name of *Entocolax rimsky-korsakovi*, in honour of the late A. P. Rimsky-Korsakov.

Diagnosis

Female. Body spindle-shaped, reaching 11.9—14.2 mm in length. In sex-mature individuals pseudopallium swollen in the form of a vesicle. Siphon not differentiated into parts. Visceral sac relatively large. Short oesophagus shows radial glandular protrusions. Intestinal sac differentiated into lobes. Sexual system composed of a branching ovary, an oviduct and an uterus. Receptaculum seminis present. The genital opening is situated on the lateral surface of the visceral sac. A single statocyst present.

Male. Dwarfish, rudimentary, about 0.4 mm across. Body in the shape of a thin-walled vesicle, with a round testicule inside. The length of the seminal duct is equal to that of the diameter of the testicule. Other organs represented by amorphous rudiments only. Located in the brood chamber of the female.

Larva. Much like that of *E. schwanwitschi*, possesses a shell, a mantle, a cephalic lobe, a foot with pedal glands, and an operculum. Nervous ganglia, statocysts and a sex rudiment present.

Parasitic of the body cavity of *Myriotrochus mitsukuri*.

Locality. Sea of Japan, Bay of Peter the Great, south of Cape Povorotny, A. W. Iwanow, 2. X. 1931, 2 specimens. South to Cape Sredny, A. W. Iwanow, VI. 1932, 1 specimen.

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Received
19.VI.1945.

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